



As we wait for others to join, start playing and exploring with the Cuisenaire rods.
Use the Zoom 'Annotate' feature to generate questions about the rods here.



Cuisenaire Rods: Endless Possibilities!

Dr. Lauren Schiller

lauren.schiller@tc.columbia.edu



AGENDA

- Introductions
- Question Generation about Rods
- Brief Background on Research
- Connecting Rods to Number Lines
- Estimation
- Place Value
- Addition, Subtraction, Multiplication, Division
- Fractions
- Lesson Planning



Whole numbers

Fractions

Number lines are very important!!

Percentages

Decimals

Integers

(Fuchs et al., 2013; Fuchs et al., 2014; Saxe et al., 2013; Rittle-Johnson et al., 2001; Schneider et al., 2009; Moss & Case, 1999)



Type of Magnitude and Main Acquisition Period

Small whole numbers (\approx 3 to 5 years)



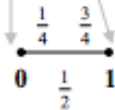
Larger whole numbers (\approx 5 to 7 years)



Yet larger whole numbers (\approx 7 to 12 years)



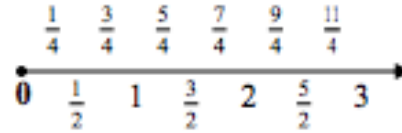
Fractions 0-1 (\approx 8 years to adulthood)



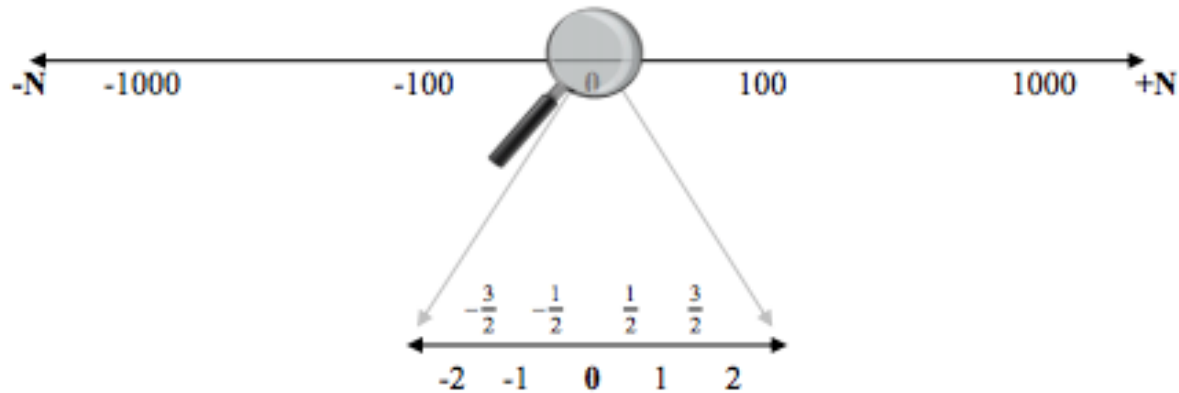
(Siegler, 2016)



Fractions 0-N (\approx 11 years to adulthood)



Rational numbers (including negatives) (\approx 11 years to adulthood)



(Siegler, 2016)



Integrated Theory of Numerical Development



**Siegler, Thompson, &
Schneider, 2011**





Early Predictors of High School Math Achievement

Siegler et al., 2012

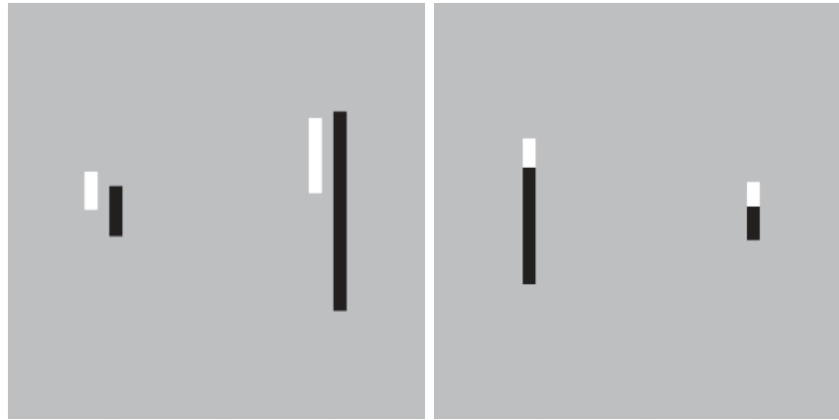


$$\begin{array}{r} 26 \\ 4 \overline{) 104} \\ \underline{-0} \\ 10 \\ \underline{-8} \\ 24 \\ \underline{-24} \\ 0 \end{array}$$



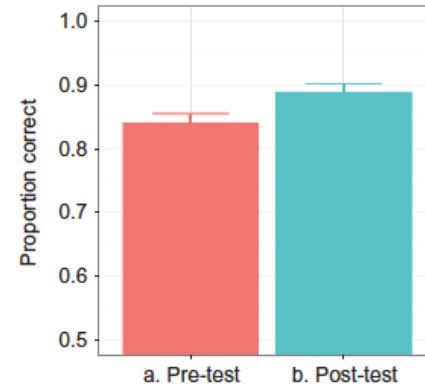
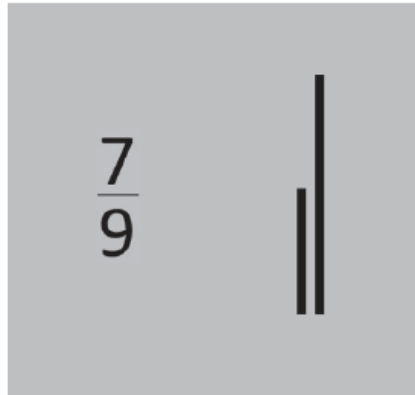
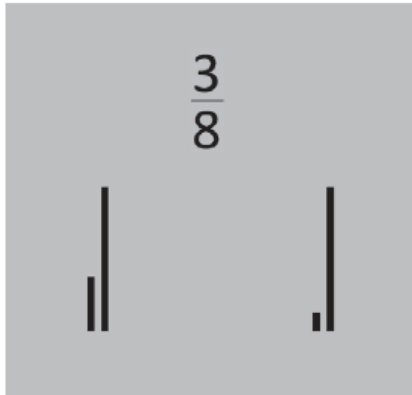


Non-symbolic Ratio Processing System (RPS) predicts fraction and algebra knowledge Matthews et al., 2016; Lewis et al., 2016



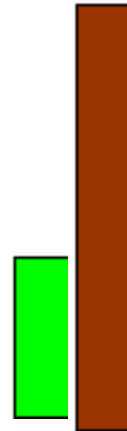
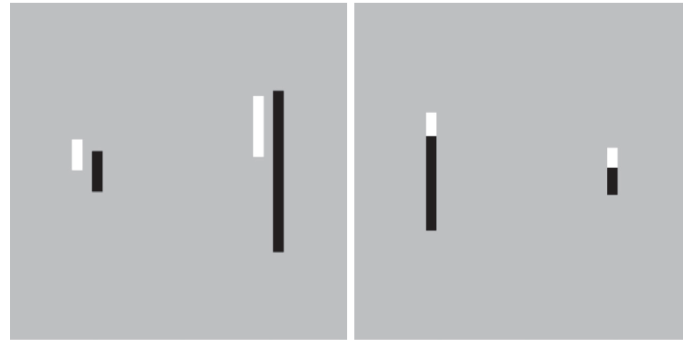


Non-symbolic Ratio Processing System (RPS) predicts fraction and algebra knowledge Matthews et al., 2016; Lewis et al., 2016









Do these remind you of anything?



From non-symbolic to symbolic proportions and back: a Cuisenaire rod proportional reasoning intervention enhances continuous proportional reasoning skills

January 2021

DOI: [10.31234/osf.io/tc8af](https://doi.org/10.31234/osf.io/tc8af)

 Roberto A. Abreu-Mendoza ·  Linsah Coulanges ·  Kendell Ali · [Show all 5 authors](#) ·  Miriam Rosenberg-Lee



**Arrange in Order,
Shortest to Longest**



number of units



Did you make a staircase?

Can you label them?



number of units

1 (white)



2 (red)



3 (lime green)



4 (purple)



5 (yellow)



6 (dark green)



7 (black)



8 (brown)



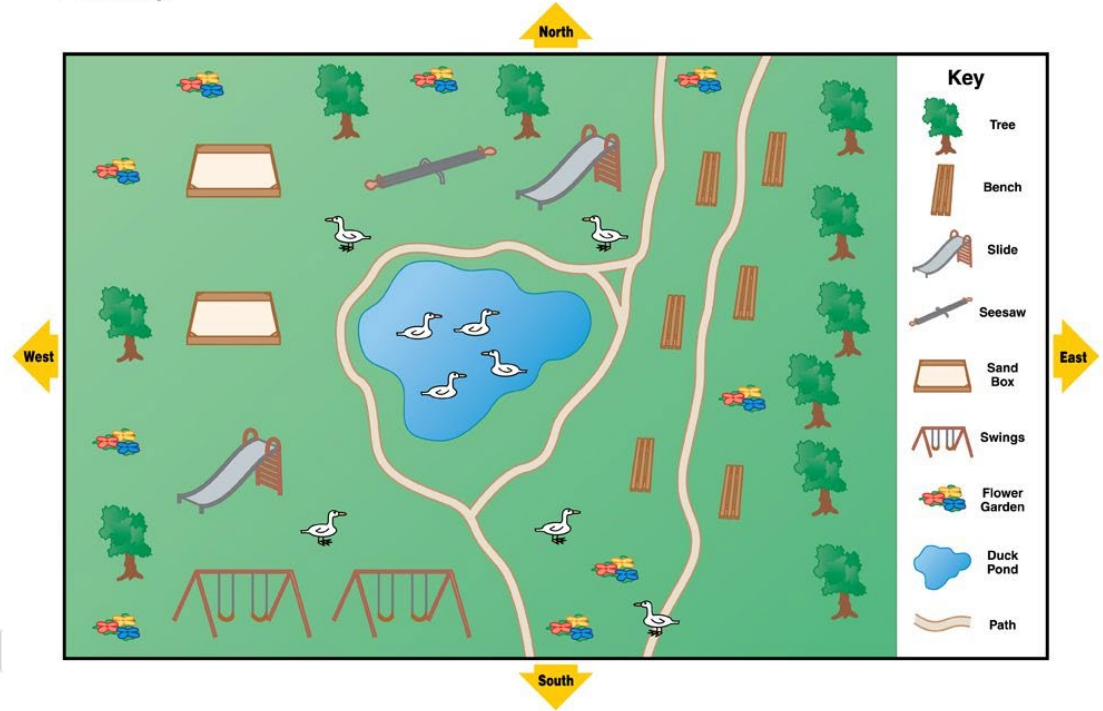
9 (blue)



10 (orange)



Park Map





Grab Bag

Game 1: 1 of each color

Game 2: 2 of each color

Game 3-4: 3 of each color

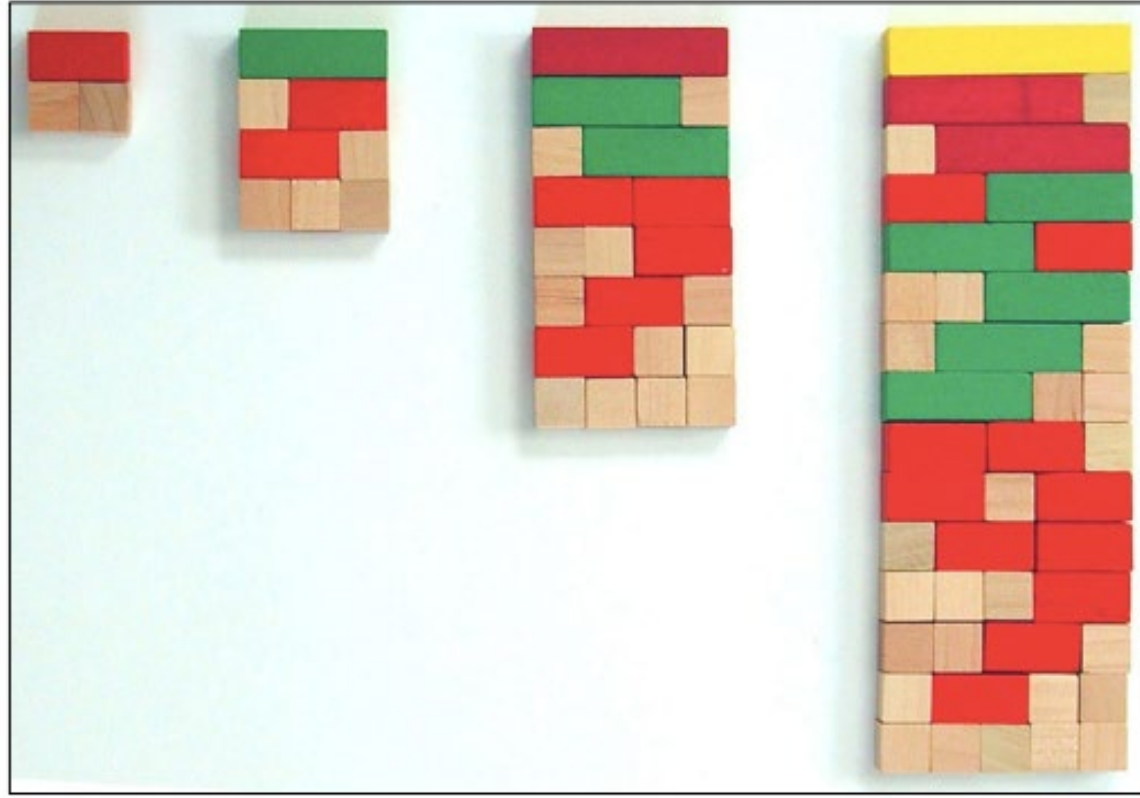
Game 5: Students pick 10-30 rods to put in the bag



Make Trains Equivalent to....
Yellow
Dark Green
Black

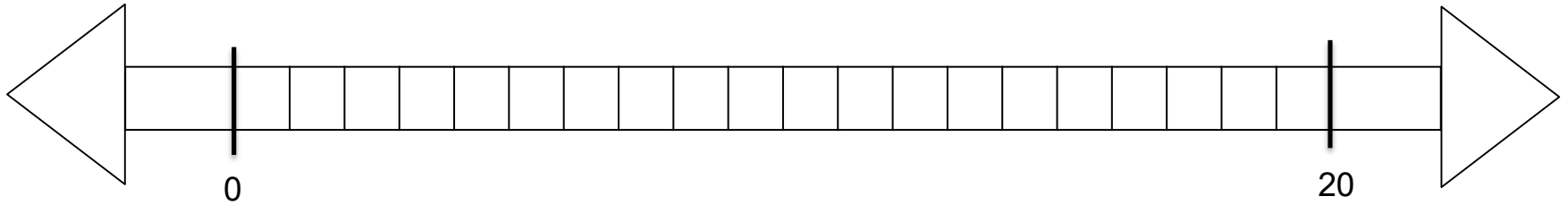
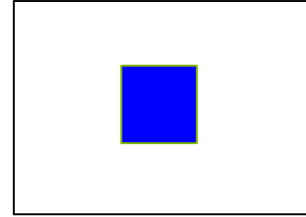


<https://nrich.maths.org/4348>

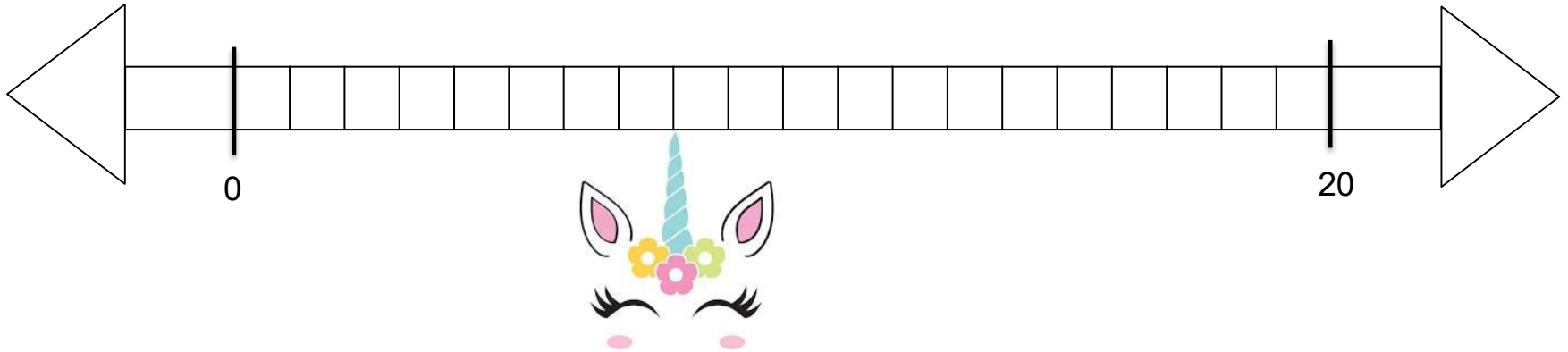
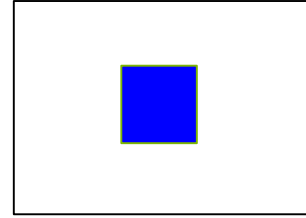




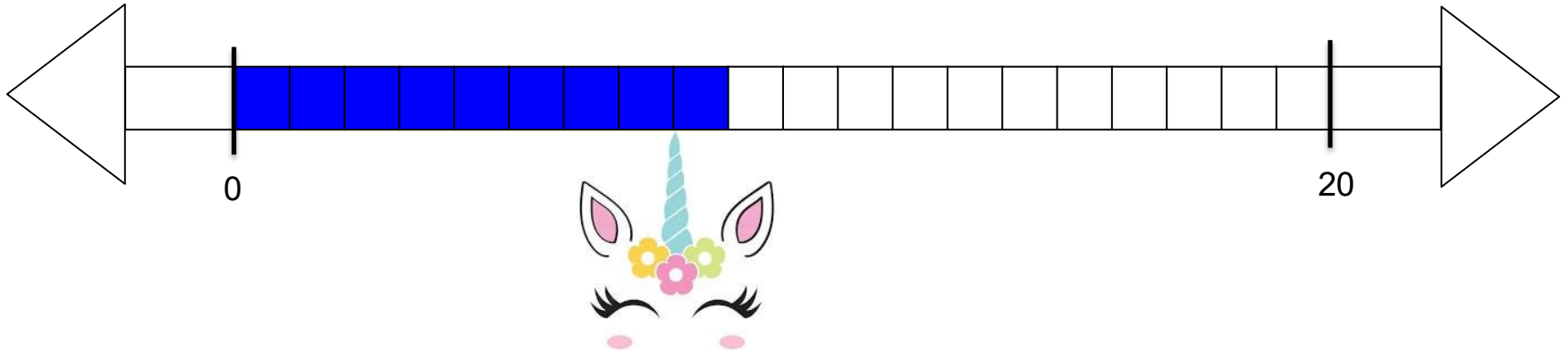
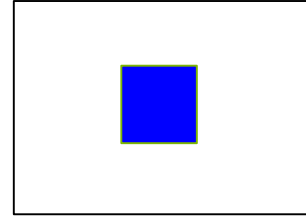
Non-Symbolic Number Line Estimation: Addition



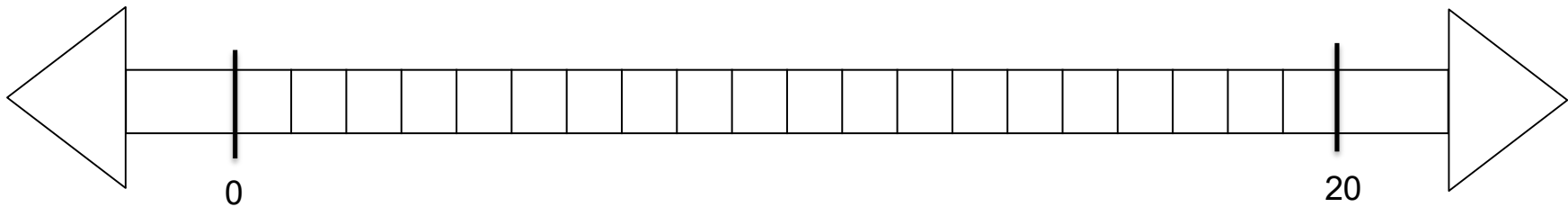
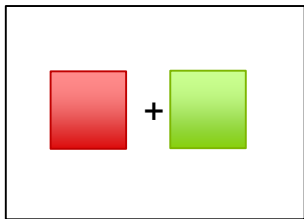
Number Line Estimation



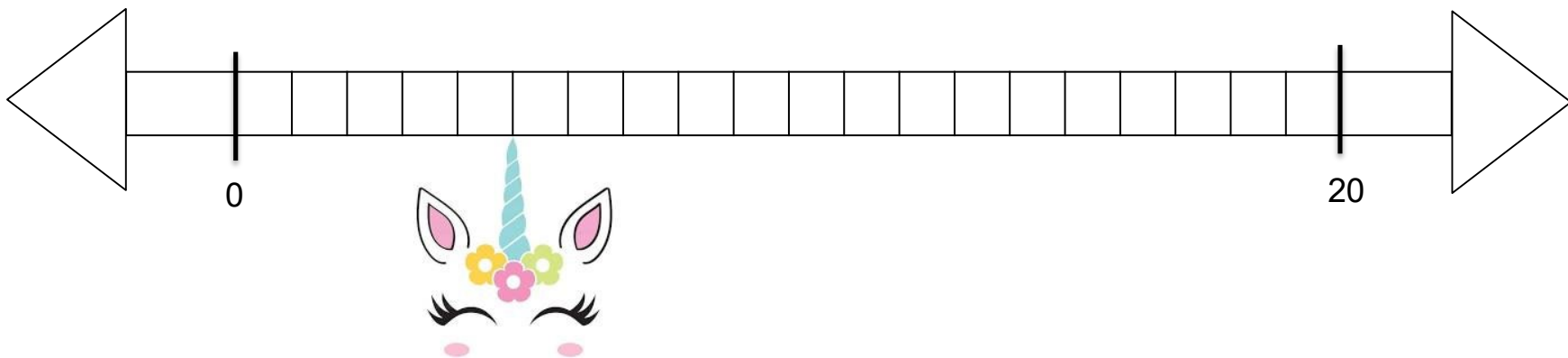
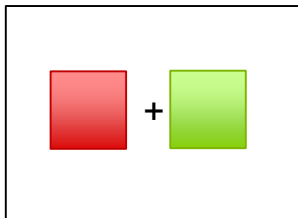
Number Line Estimation



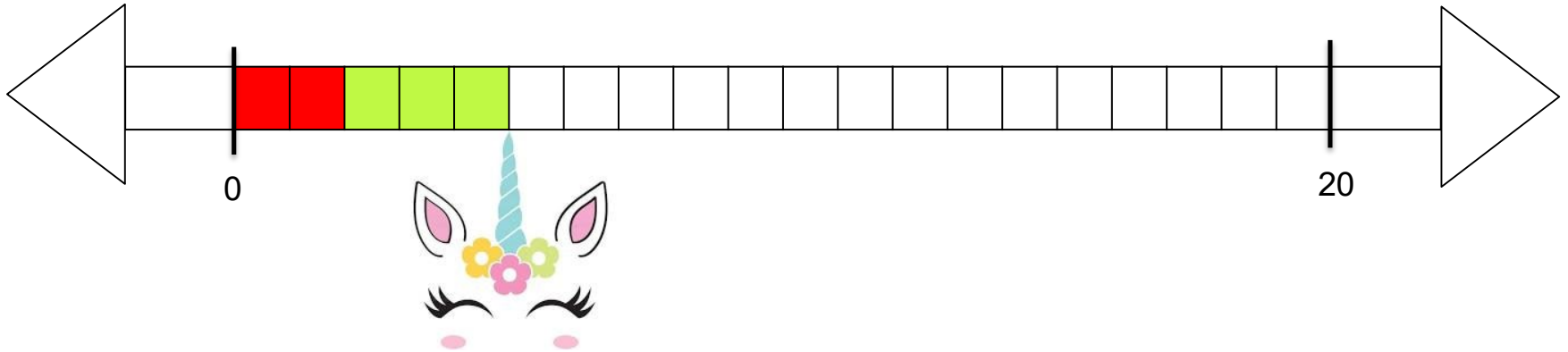
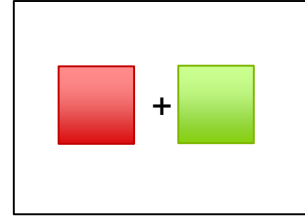
Number Line Estimation



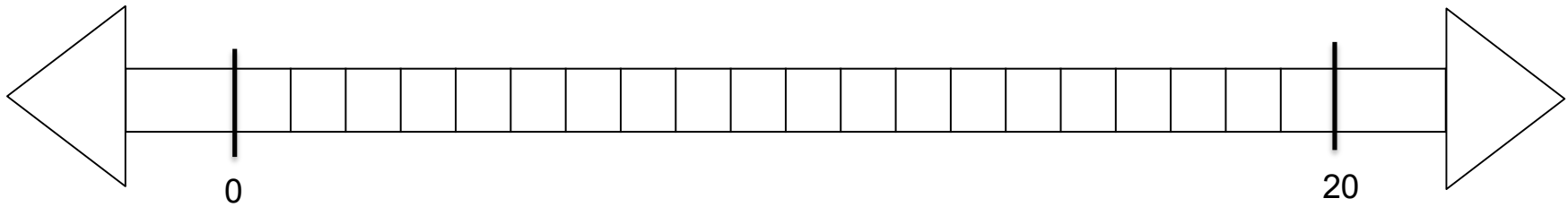
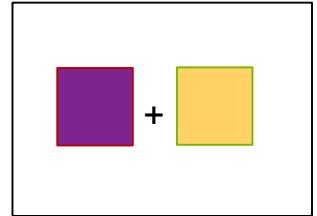
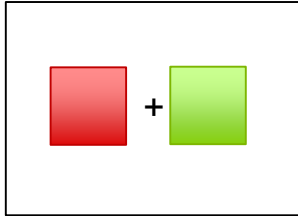
Number Line Estimation



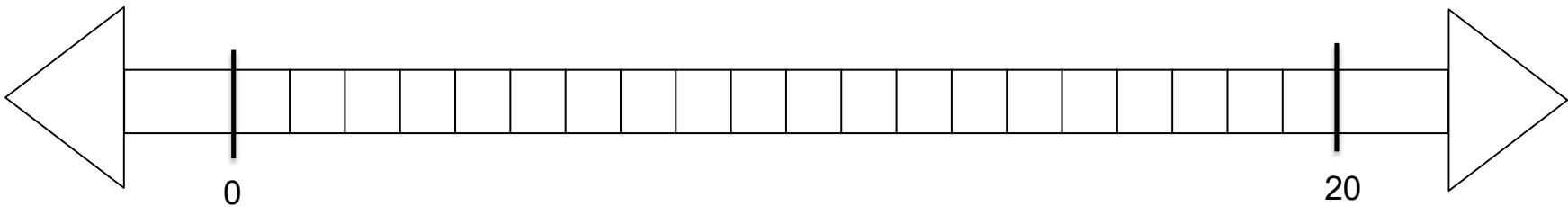
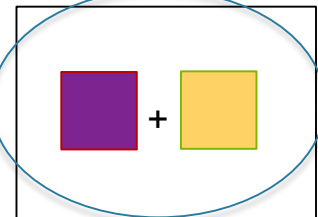
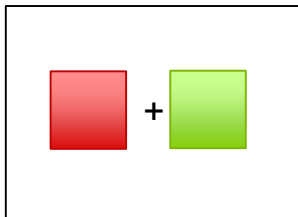
Number Line Estimation



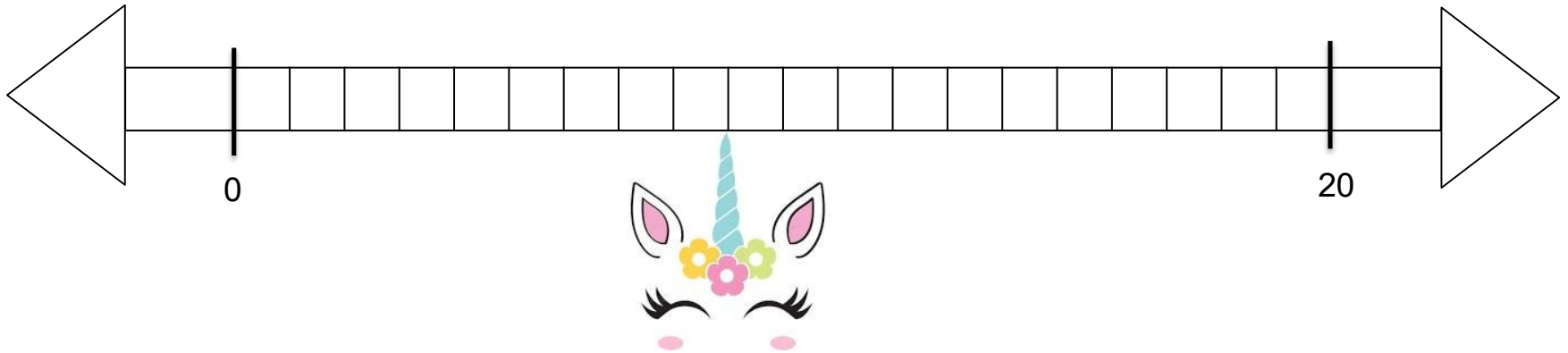
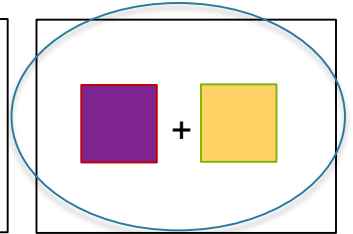
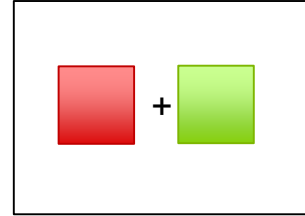
Number Line Estimation



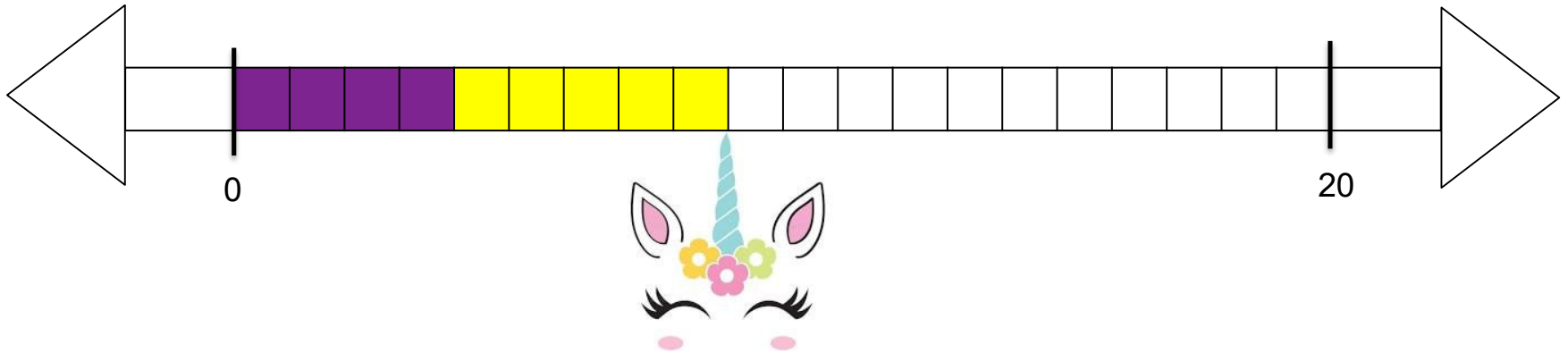
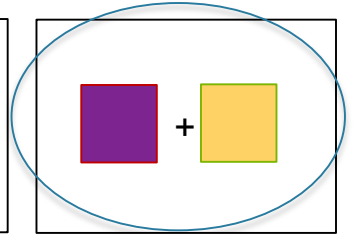
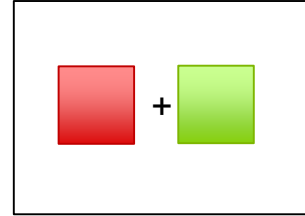
Number Line Estimation



Number Line Estimation



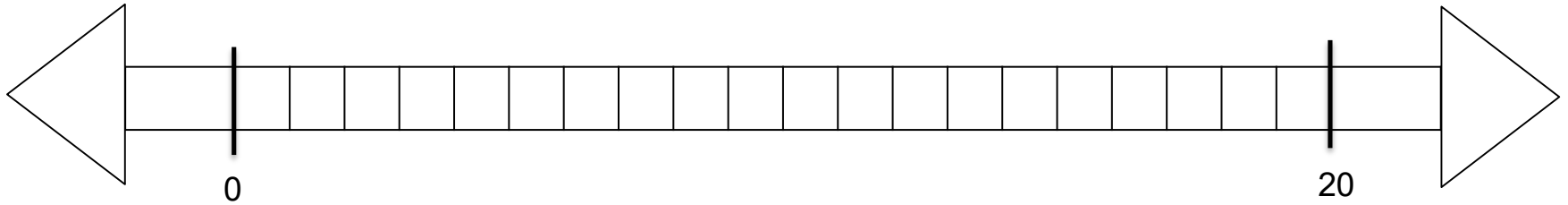
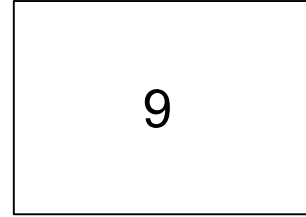
Number Line Estimation



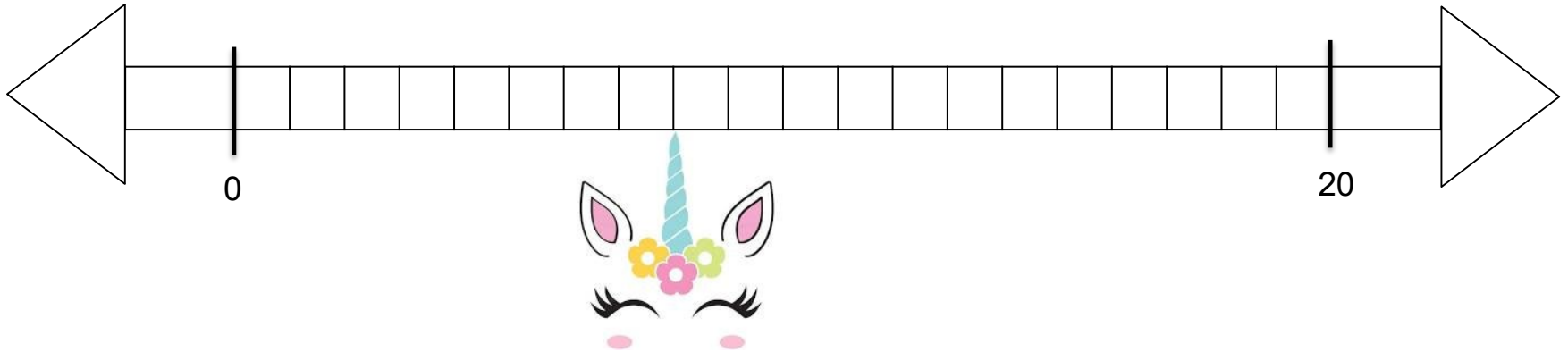
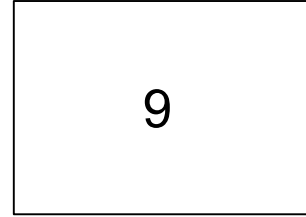
Number Line Estimation



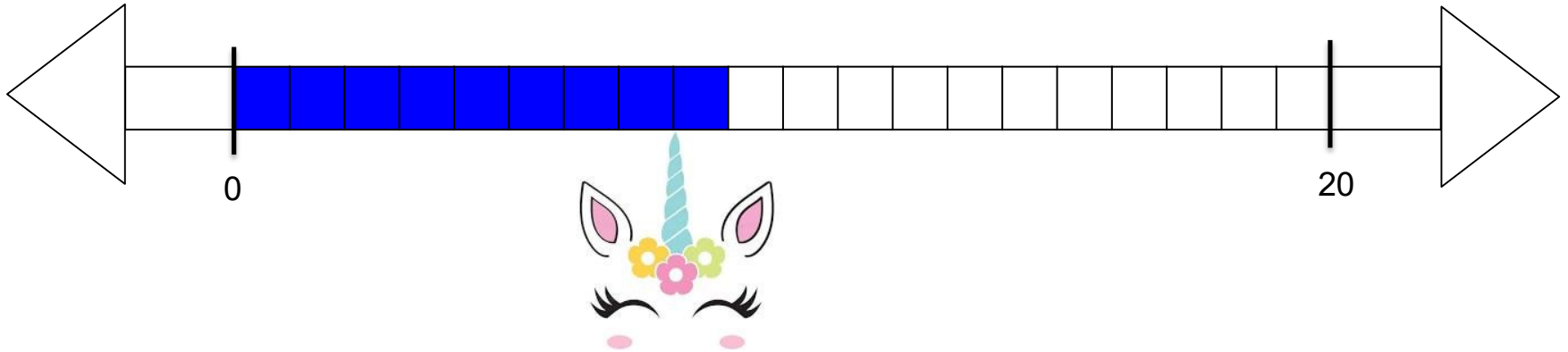
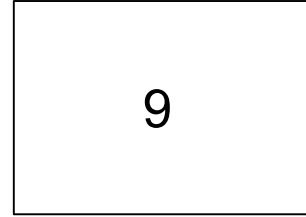
Symbolic Number Line Estimation: Addition



Number Line Estimation



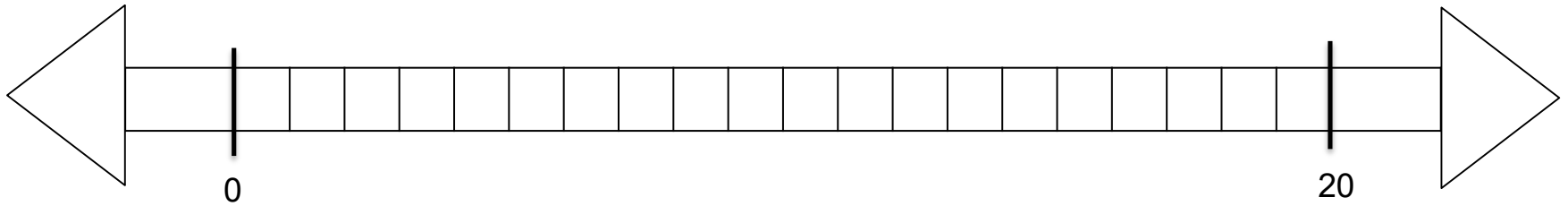
Number Line Estimation



Number Line Estimation



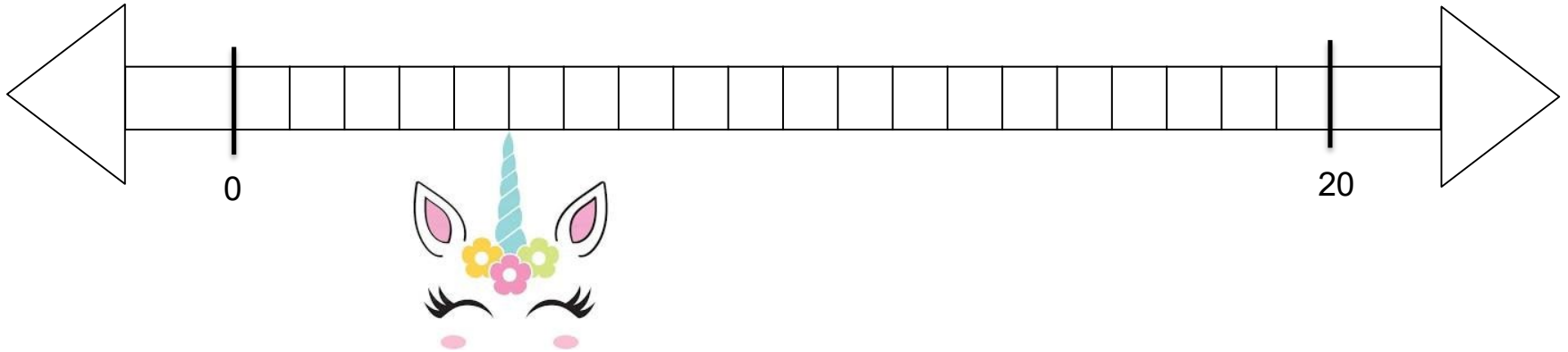
$$2 + 3$$



Number Line Estimation



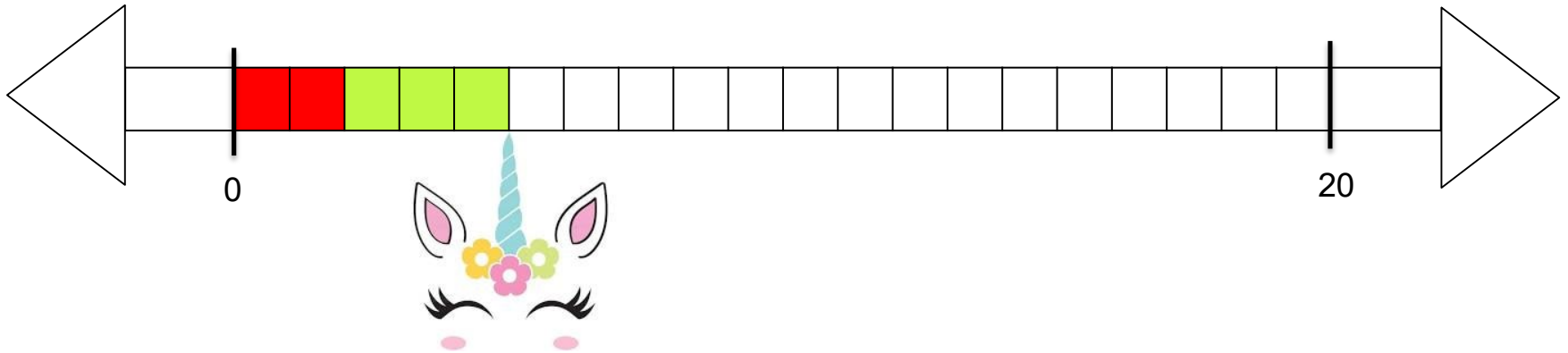
$$2 + 3$$



Number Line Estimation



$$2 + 3$$

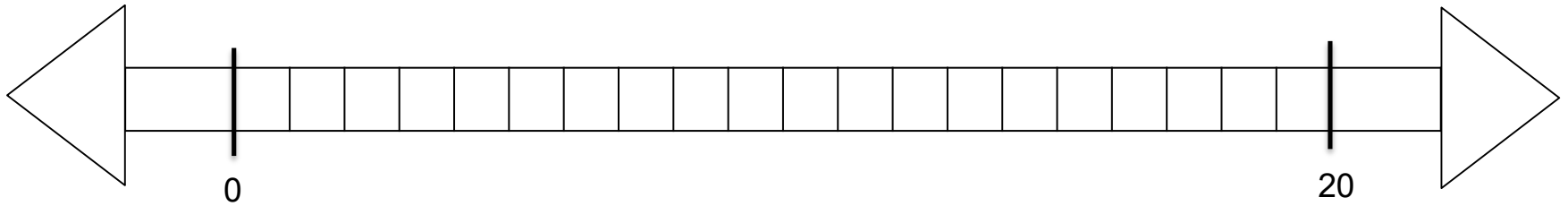


Number Line Estimation



$$2 + 3$$

$$4 + 5$$

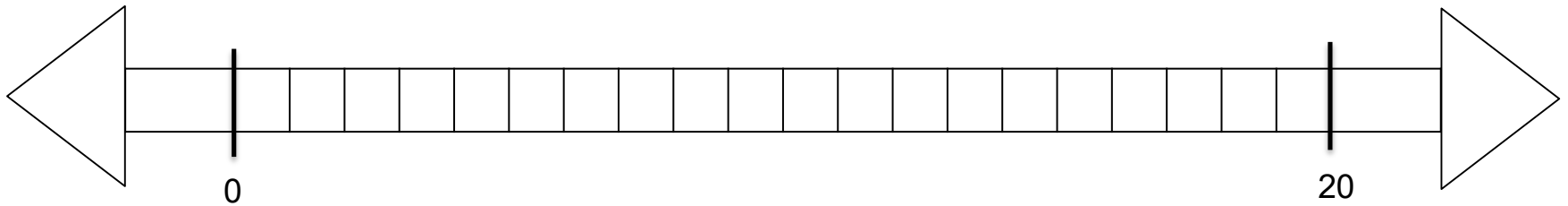


Number Line Estimation



$$2 + 3$$

$$4 + 5$$

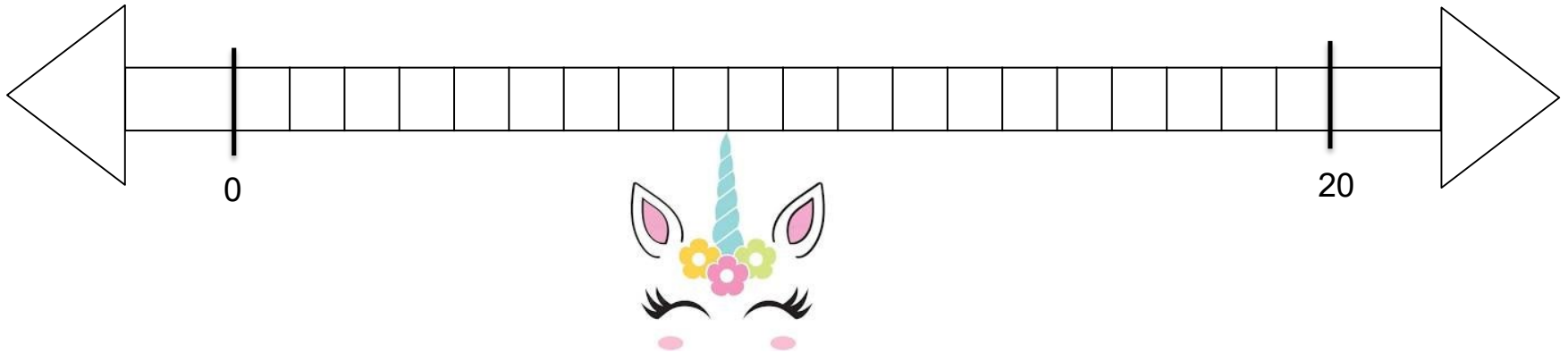


Number Line Estimation



$$2 + 3$$

$$4 + 5$$

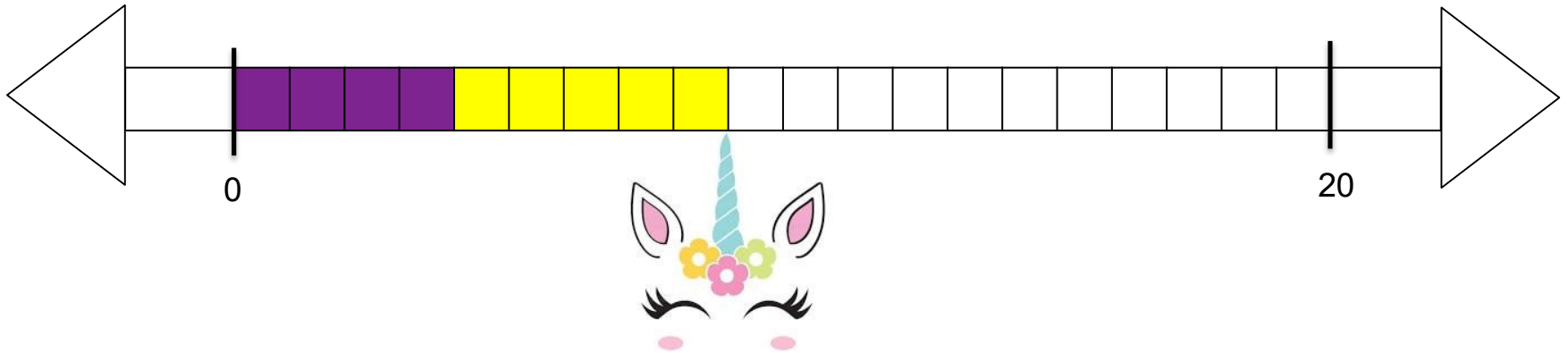


Number Line Estimation



$$2 + 3$$

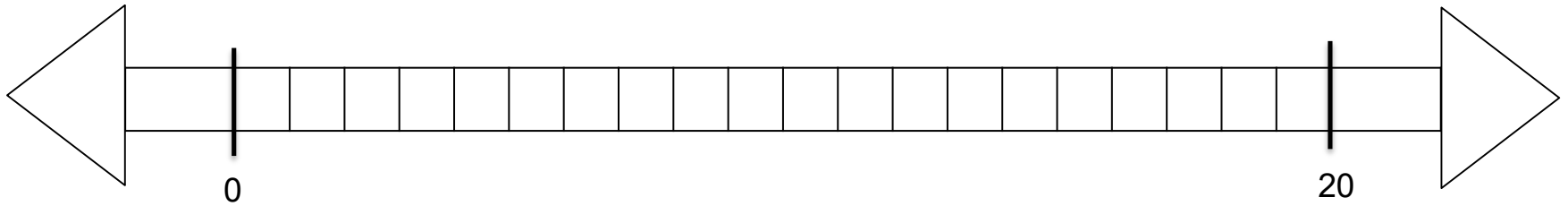
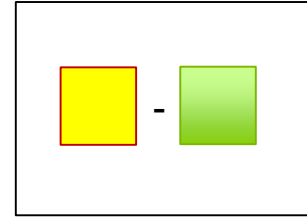
$$4 + 5$$



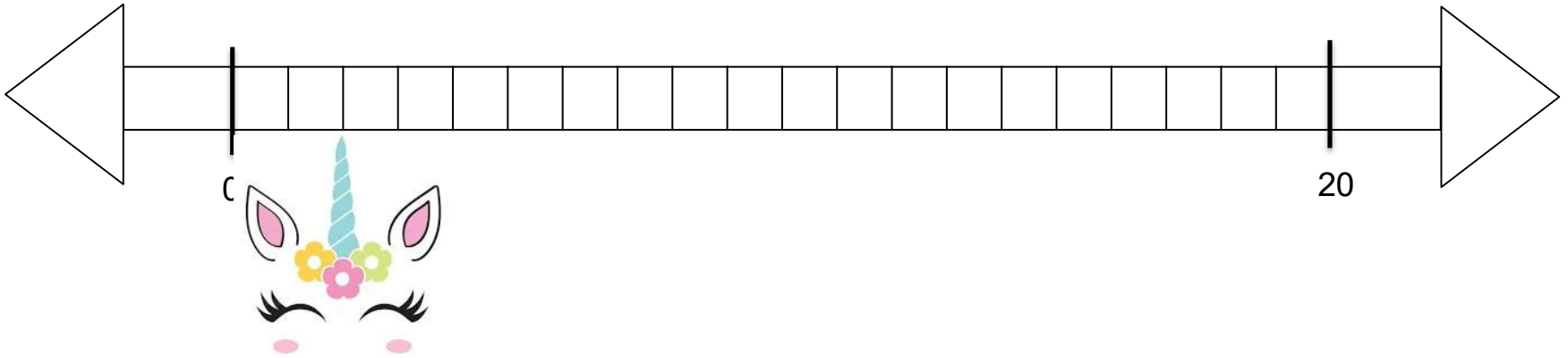
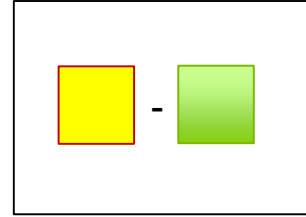
Number Line Estimation



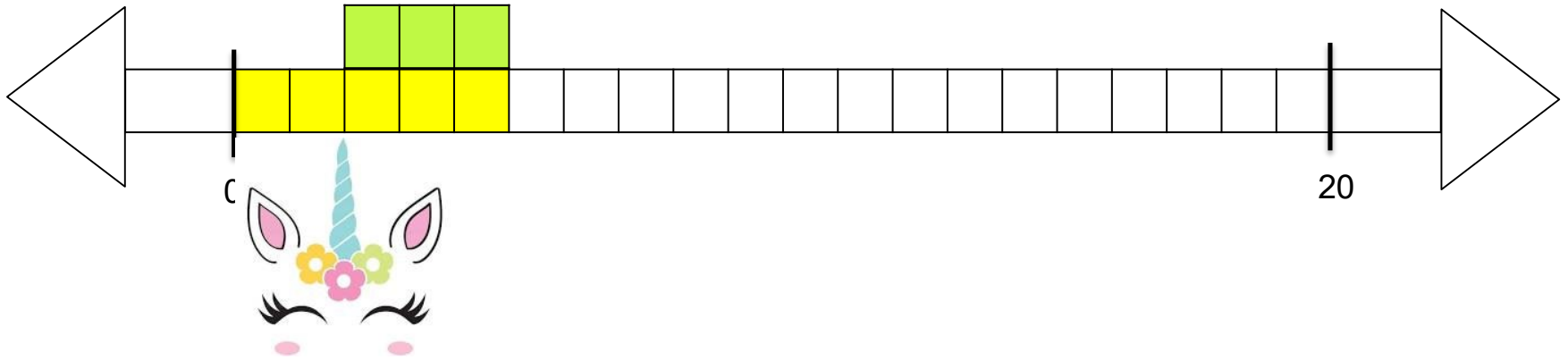
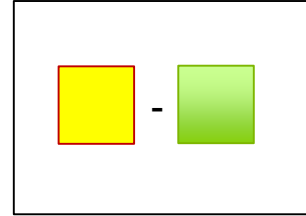
Non-Symbolic Number Line Estimation: Subtraction



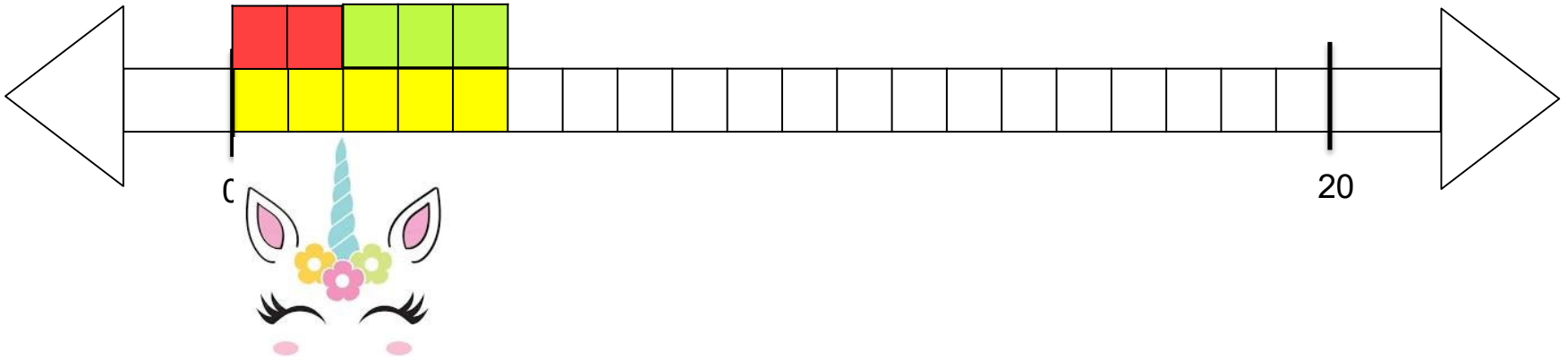
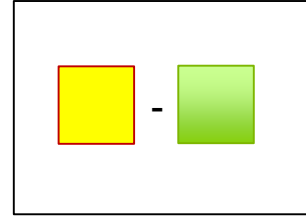
Number Line Estimation



Number Line Estimation



Number Line Estimation



Number Line Estimation



Make 10

<https://nrich.maths.org/4348>



Go Fish!

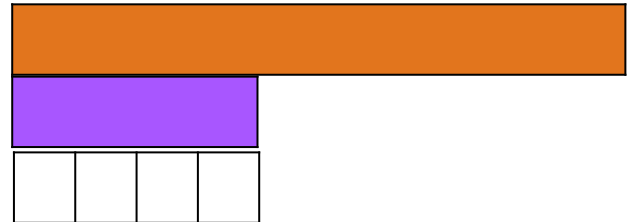


Go Fish!

14





=

10+4





Place Value

Thousands	Hundreds	Tens	Ones
1	2	3	4
			



Try some addition problems

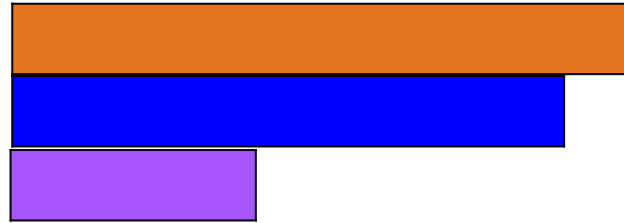
$$19+4$$

$$39+13$$



Try some addition problems

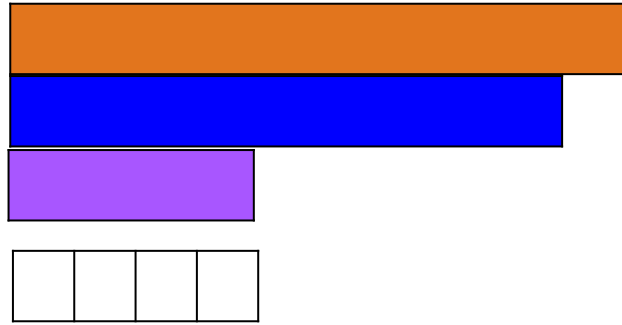
19+4





Try some addition problems

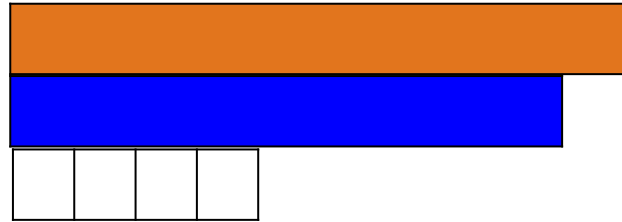
$$19+4$$





Try some addition problems

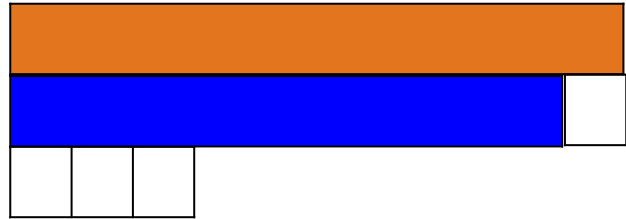
$$19+4$$





Try some addition problems

$$19+4$$

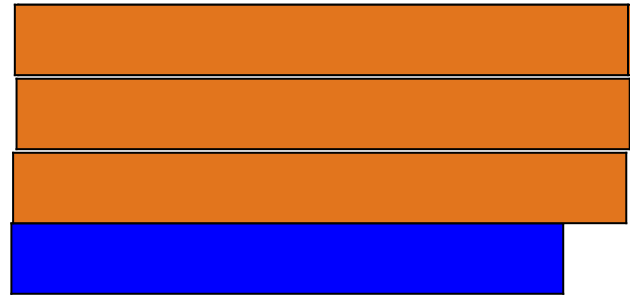


$$\begin{array}{r} 1 \\ 19 \\ + 4 \\ \hline 23 \end{array}$$



Try some addition problems

$$39 + 13$$





Try some addition problems

$$39 + 13$$

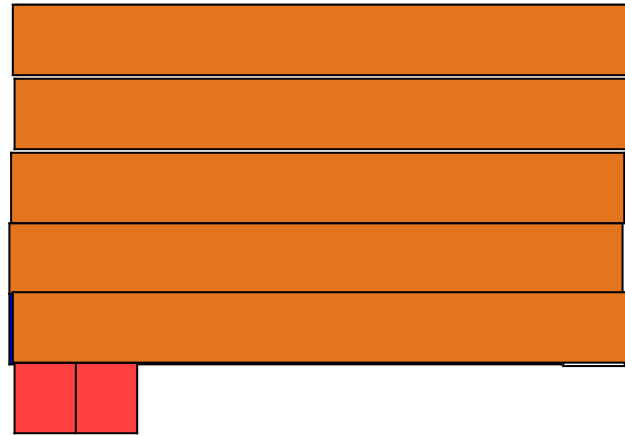




Try some addition problems

$$39+13$$

$$\begin{array}{r} 1 \\ 39 \\ +13 \\ \hline 52 \end{array}$$





Try some subtraction problems

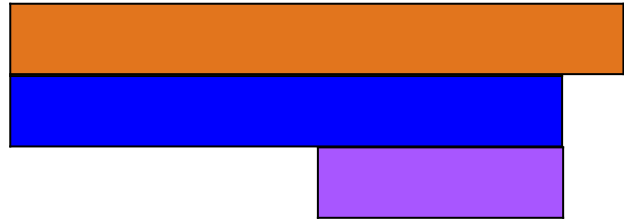
$$19-4$$

$$33-19$$



Try some subtraction problems

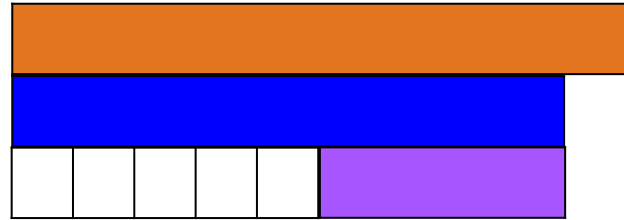
19-4





Try some subtraction problems

$$19 - 4$$

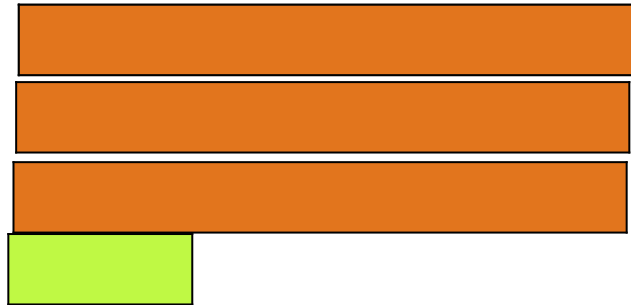


$$\begin{array}{r} 19 \\ - 4 \\ \hline 15 \end{array}$$

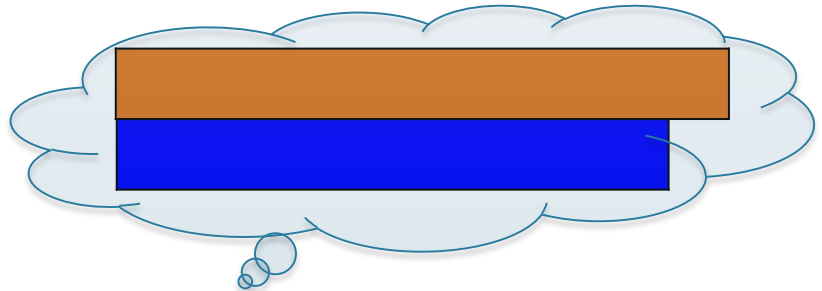


Try some subtraction problems

33-19



33
-19





Try some subtraction problems

33-19



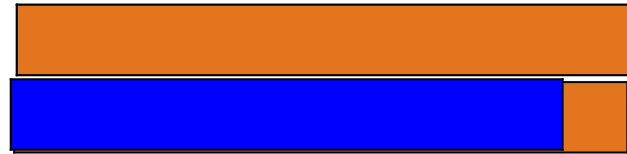
2 13
~~33~~
-19
14





Try some subtraction problems

33-19



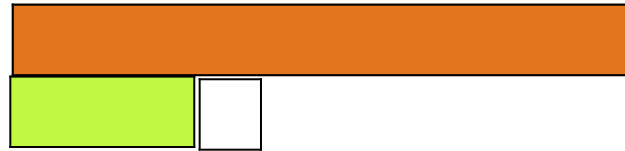
2 13
~~33~~
-19
14





Try some subtraction problems

33-19



$$\begin{array}{r} 2 \ 13 \\ \cancel{3}3 \\ -19 \\ \hline 14 \end{array}$$



Which is longer?

- **Predict**
- **Then, Check**



Which is bigger
4 of the brown rods or
5 of the black rods?

What operation is this? Can you write a
number sentence?



4 of the brown rods
5 of the black rods

$$4 \times 8 =$$

$$5 \times 7 =$$



$$6 \times 4 =$$

$$5 \times 6 =$$

How can this be written with words
referring to rods?



$$6 \times 4 =$$

$$5 \times 6 =$$

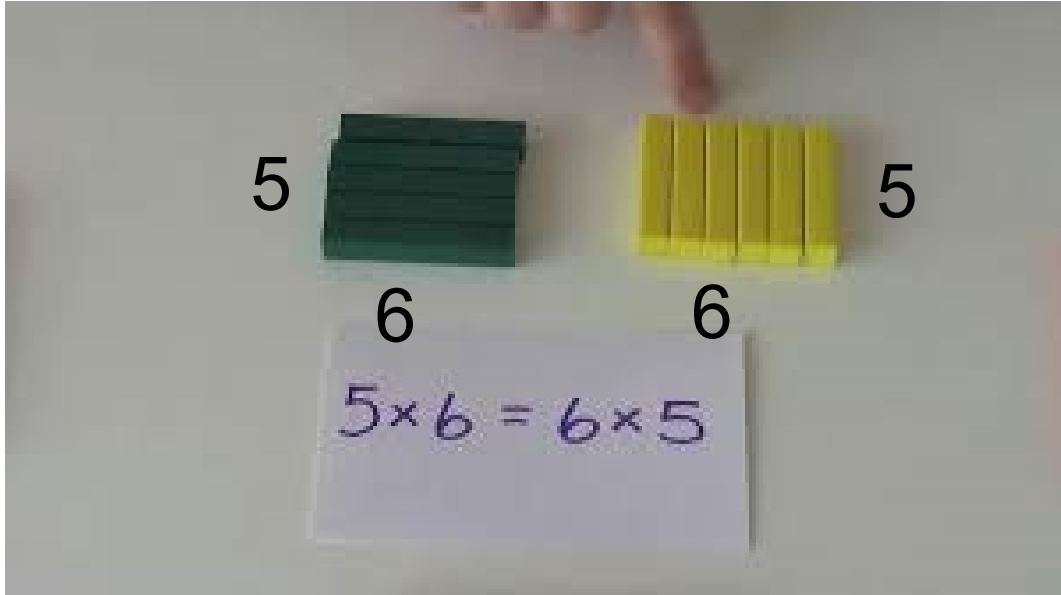
6 of purple
5 of dark green



Try some multiplication problems

5x6

6x5





Try some multiplication problems

4×7

12×2

14×8

<http://www.educationunboxed.com/multiplying-2-digit-by-1-digit-numbers/>

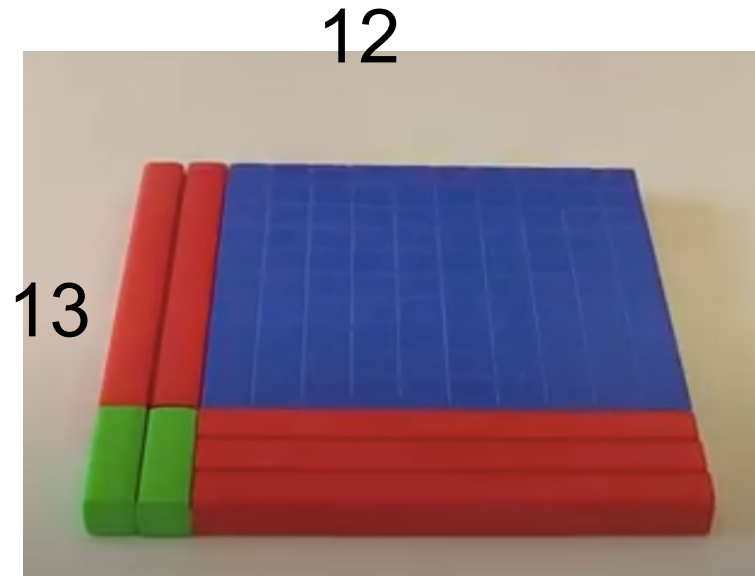


Ready for some more?

10x10

11x10

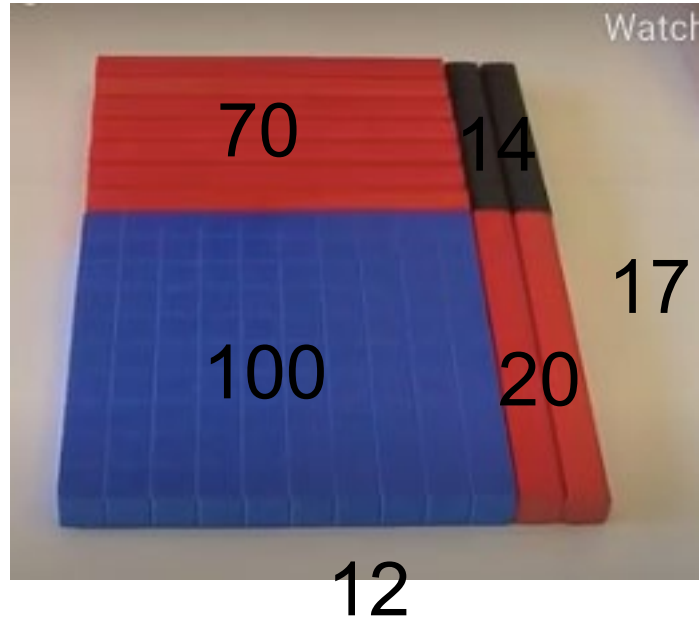
12x13



Where does the algorithm come from? Relate to the rods.



$$\begin{array}{r} 1 \\ 12 \\ \times 17 \\ \hline 84 \\ +120 \\ \hline 204 \end{array}$$



Where does the algorithm come from? Relate to the rods.

<http://www.educationunboxed.com/multiplying-large-numbers/> start at 8:04

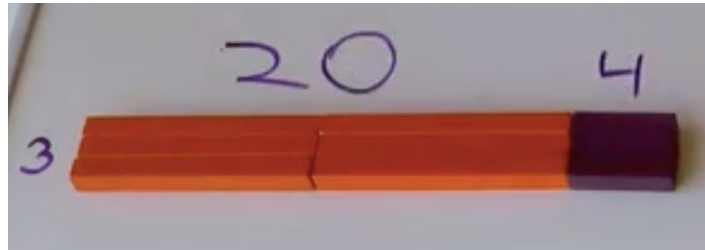


What about division?



Let's try some division!

$$72 \div 3$$



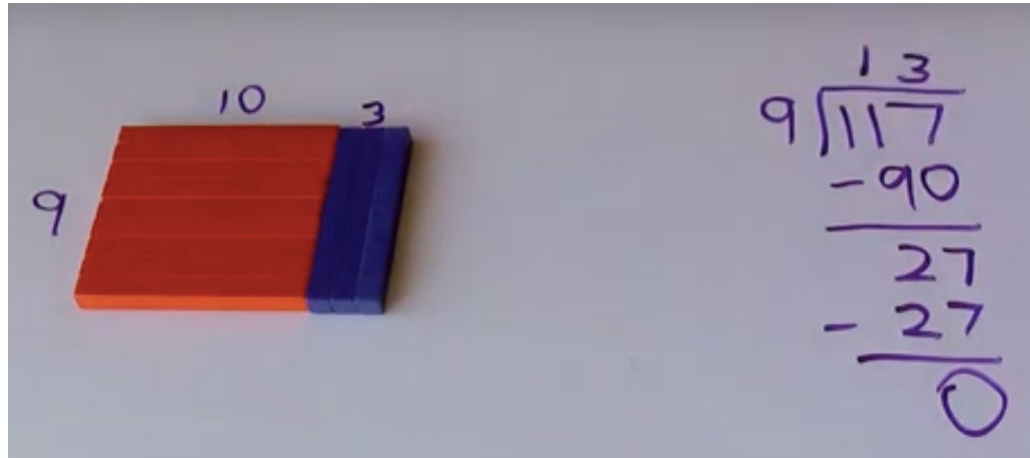
$$\begin{array}{r} 24 \\ 3 \overline{) 72} \\ \underline{-60} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

Where does the algorithm come from? Relate to the rods. <http://www.educationunboxed.com/long-division-part-one/>



Let's try some division!

$$117 \div 9$$

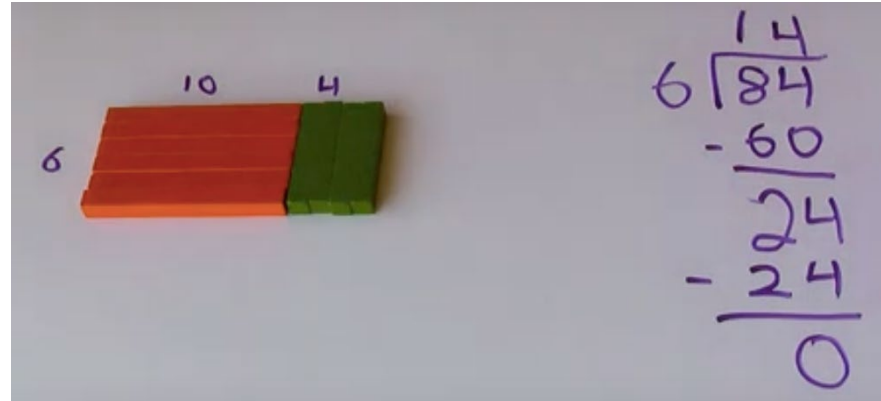


Where does the algorithm come from? Relate to the rods. <http://www.educationunboxed.com/long-division-part-one/>

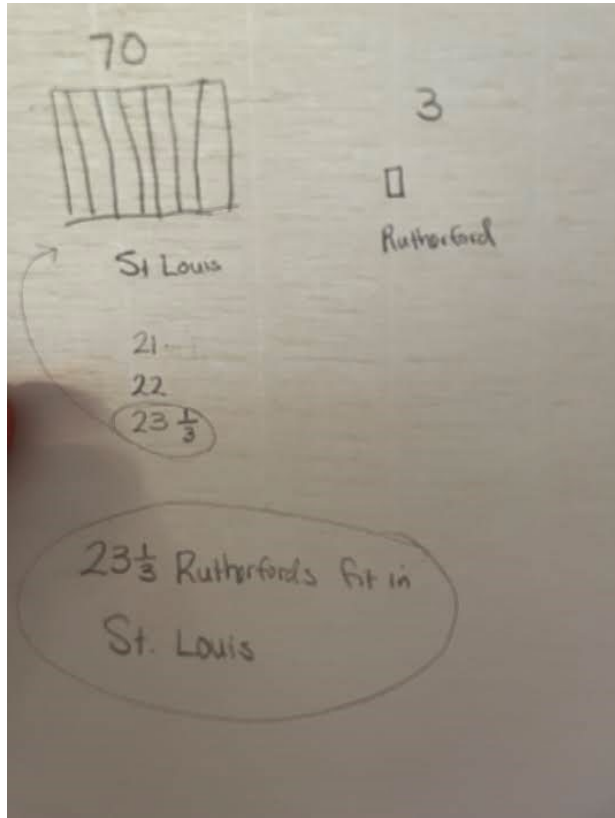


Let's try some division!

$$84 \div 6$$



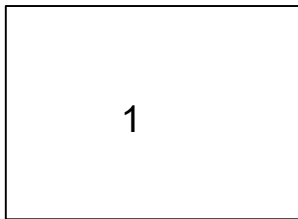
Where does the algorithm come from? Relate to the rods. <http://www.educationunboxed.com/long-division-part-one/>



How many Rutherfords would fit in St. Louis?



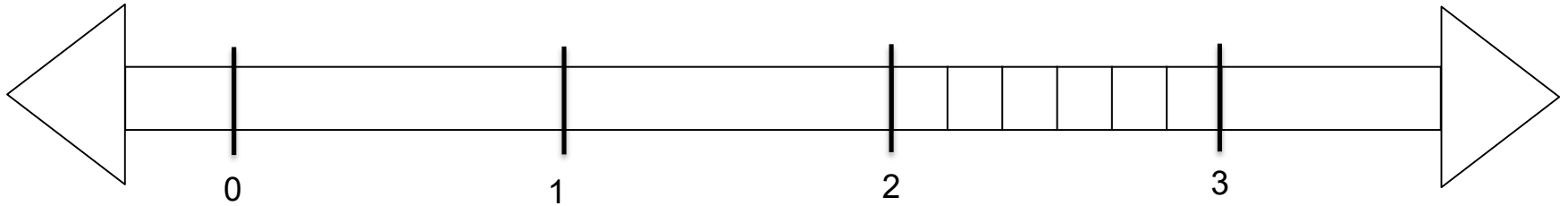
Fractions



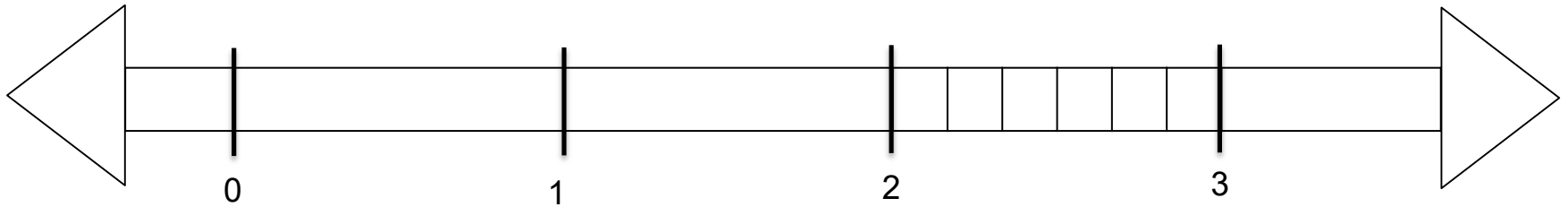
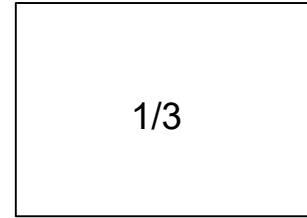
What color is 1?



1/2



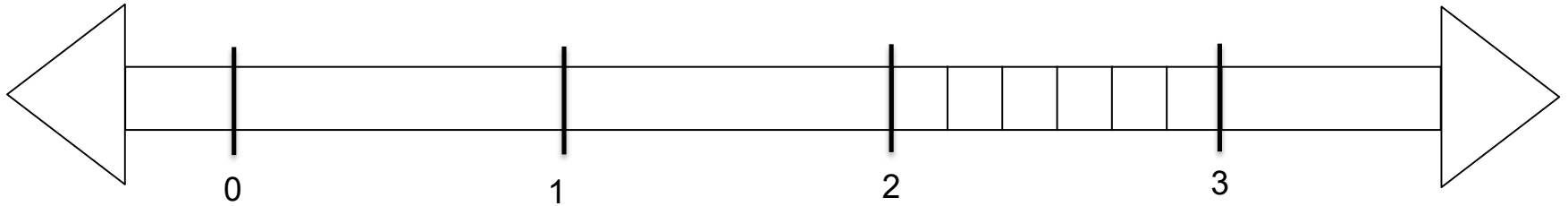
What color is 1/2?



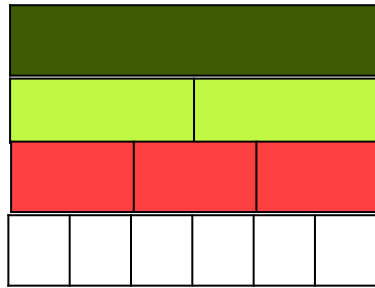
What color is $1/3$?



$1/6$

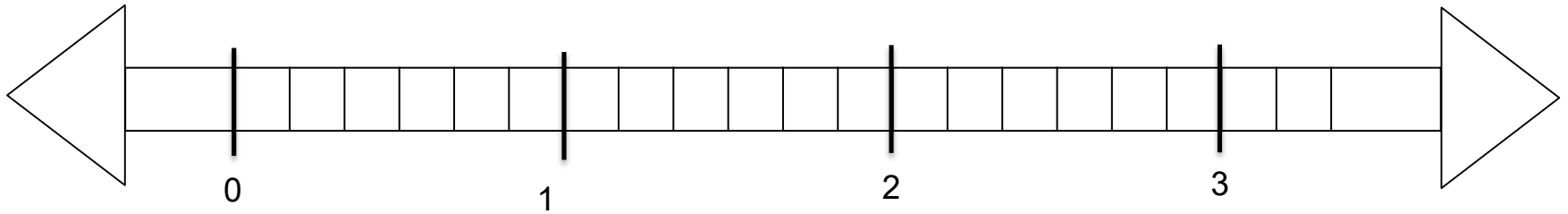
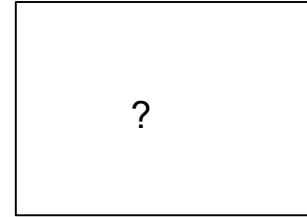


What color is $1/6$?

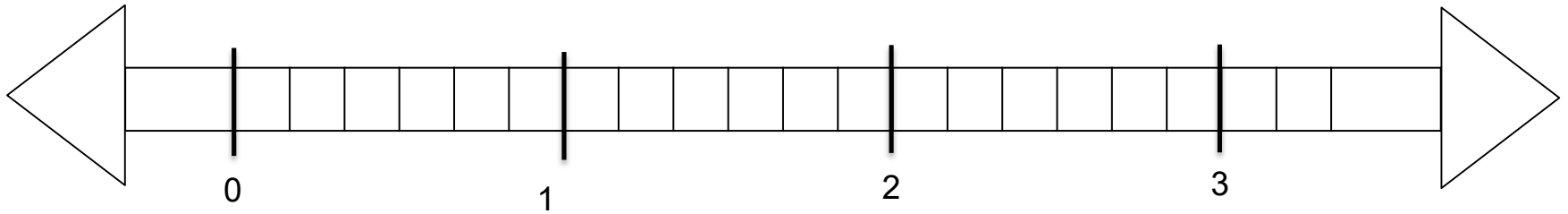
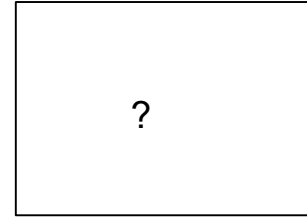




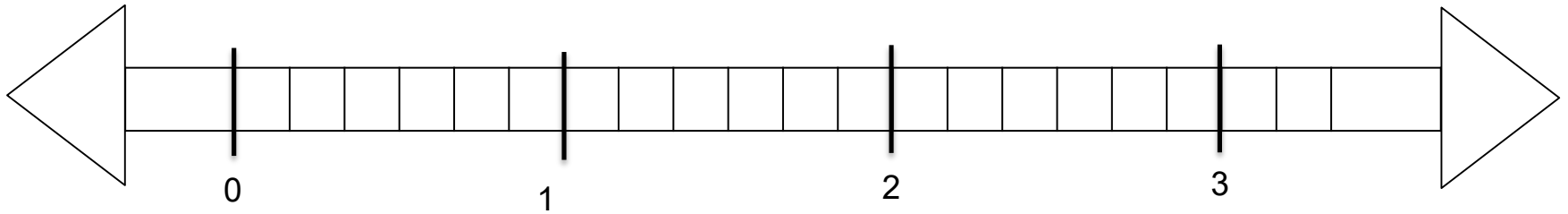
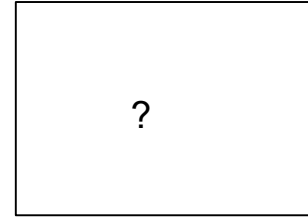
**What other fractions can you
make using the Cuisenaire
rods?**



Fraction Magnitude Representation



Fraction Magnitude Representation



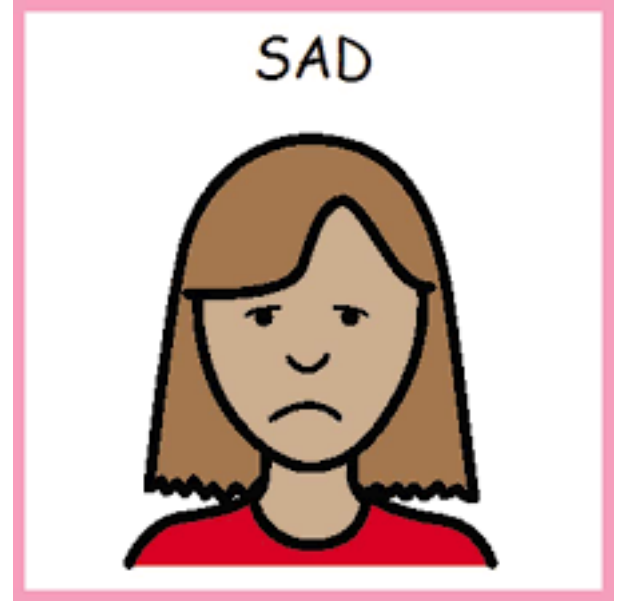
Fraction Magnitude Representation



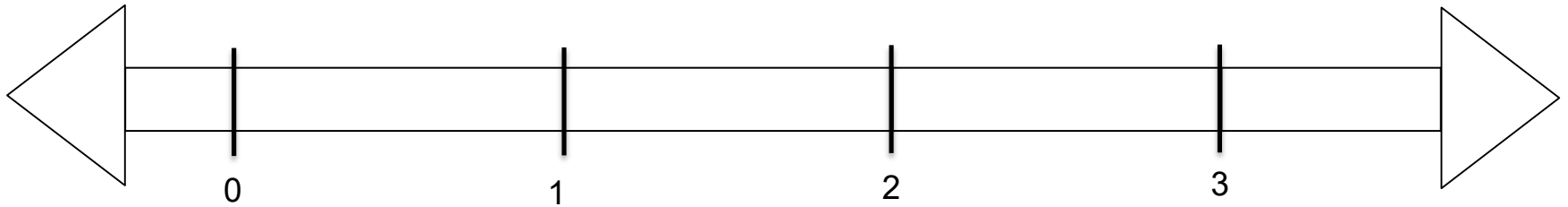
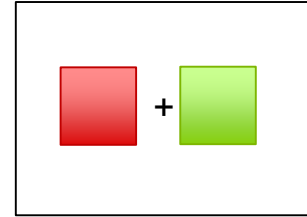
**Adding/subtracting fractions
with unlike denominators—
what do we need to
remember?**



$$\frac{1}{2} + \frac{2}{6} = \frac{3}{8}$$



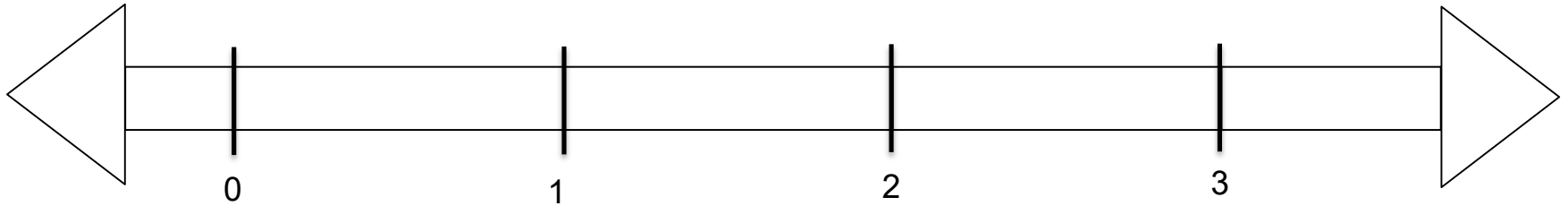
Subtraction & **A**ddition need a Common **D**enominator



Number Line Estimation



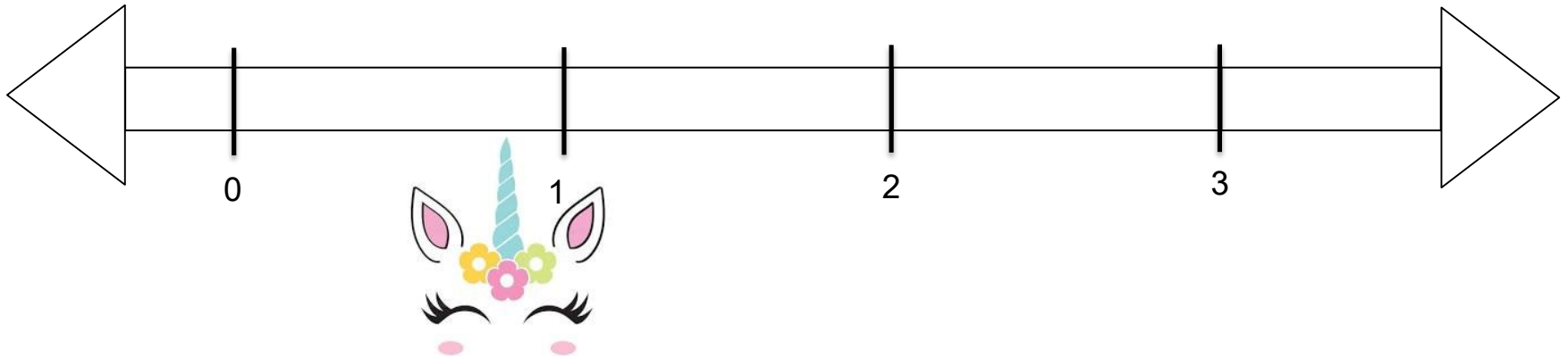
$$\frac{1}{3} + \frac{1}{2}$$



Number Line Estimation



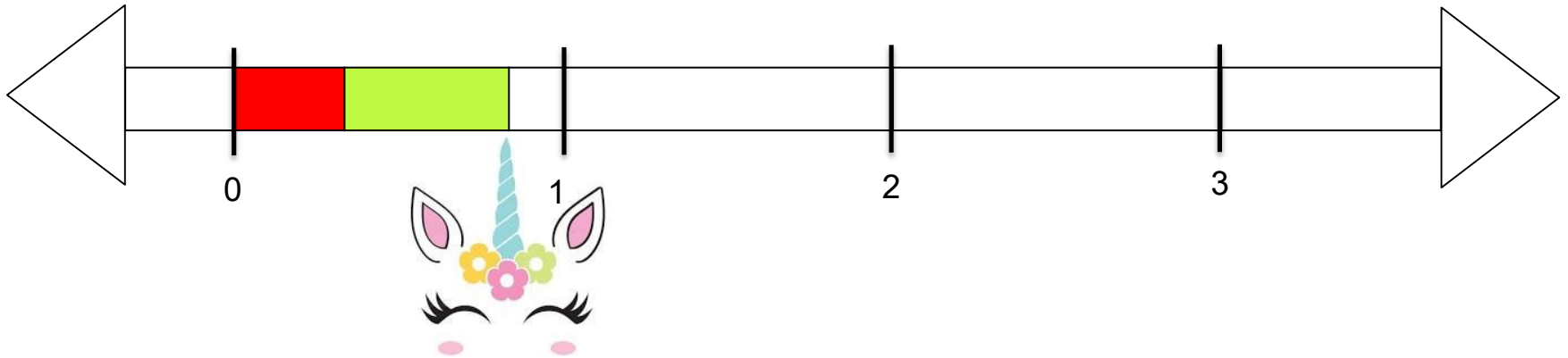
$$\frac{1}{3} + \frac{1}{2}$$



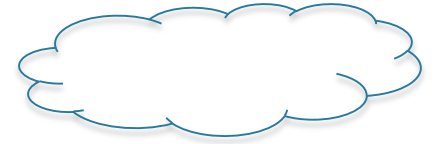
Number Line Estimation



$$\frac{1}{3} + \frac{1}{2}$$



Number Line Estimation



Numeric	Visual	Equation



A little more than 1

Numeric	Visual	Equation
$\frac{1}{3}$		
$\frac{5}{6}$		



A little less than 1

Numeric	Visual	Equation
$\frac{2}{3}$		
$\frac{1}{6}$		



Aunt Elyse's Famous Salad Dressing Recipe:

$\frac{1}{3}$ cup olive oil

$\frac{1}{6}$ cup vinegar

pinch of oregano

pinch of salt



How many cups of salad dressing will this recipe make? Write an equation to represent your thinking. Assume that the herbs/salt do not change the amount of dressing.

If this recipe makes 6 servings, **how many cups of olive oil and how many cups of vinegar will we need for 18 people?**



About half

Numeric	Visual	Equation
$\frac{1}{3}$		
$\frac{1}{6}$		



EXIT TICKET

A student solved a problem this way:

$$\frac{1}{2} + \frac{2}{6} = \frac{3}{8}$$

Explain whether the student is right or wrong and justify your reasoning.



Game/Activity Workshop

- ❑ Select a grade level standard and create a game/activity using the Cuisenaire rods
- ❑ Check out some videos on [Educationunboxed.com](https://www.educationunboxed.com) or elsewhere for Math Games/Activities
- ❑ Upload directions for your game to Padlet <https://padlet.com/lks2132/kansasrods>
- ❑ SPEED TEACH!



Thank you!

lauren.schiller@tc.columbia.edu