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Belief-Based Versus Evidence-Based Math Assessment and Instruction

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Editor's Note: The authors will expand on the ideas in this article in their convention symposium titled "Math Myth Busters: Debunking Common Misunderstandings With MTSS Implementation."

Many school psychologists work in schools that have low proficiency rates on the year-end test of mathematics, which is concerning because math proficiency is a powerful indicator of long-term academic success. For example, Duncan et al. (2007) found that early numeracy measures forecast later academic proficiency even better than early literacy measures among young students. Children who meet the college-readiness benchmarks in mathematics tend to complete 2-year and 4-year degrees at a higher rate and experience higher lifetime earnings (Lee, 2012). Assisting schools to help all students meet the college-readiness benchmarks in math performance is a socially meaningful action that, in effect, can be an economic gateway to their future lives.

Fortunately for school psychologists, identifying which students are on track toward the college-readiness benchmarks is knowable given certain milestone indicators, like early numeracy mastery by kindergarten, whole number operation mastery by grade 3, proportion quantity and operation mastery by grade 6, and linear function mastery by grade 8. Unfortunately, many systems fail to notice lagging progress until children fail to master proportions, by which time children are sorted into math tracks (e.g., advanced math, remedial math). Being sorted into a remedial track at grade 7 or 8 when lagging performance could have been addressed and prevented in grades 1 to 5 leads many children to unfairly miss out on the lifetime economic benefit of mathematical proficiency (Lee, 2012). In fact, a recent study found that fifth grade math proficiency was the strongest basis for predicting who would meet the ACT college readiness benchmark for math, not the sequence of instruction followed in secondary schooling (e.g., remedial versus advanced math course sequences; Koon & Davis, 2019).

Addressing math instructional problems is not easy work. One reason that the work is so challenging for school psychologists is that there is a great deal of philosophy that is at odds with contemporary evidence, yet is embraced by the teachers, school coaches, and leaders who observe the large and persistent achievement gaps, care deeply about their students, and want to avoid harm. The result is tension between evidence-based and philosophy-based practices in math education. Although consensus building documents, such as Adding It Up (National Research Council [NCR], 2001) and the National Mathematics Advisory Panel Report (NMAP, 2008), have been published, conflicting recommendations around key approaches to mathematics instruction and intervention are promoted through websites, blogs, and formal organizations (Doabler et al., 2015; Rittle-Johnson, Schneider, & Star, 2015). This wealth of information circulating in both traditional and modern outlets makes it difficult to distinguish between pseudoscience and scientific approaches to quality mathematics instruction and intervention (Kratochwill, 2012; Lilienfeld, Ammirati, & David, 2012) and ultimately leaves students vulnerable to ineffective approaches by well-meaning adults. We believe that if school psychologists can understand the origin of this tension, they can be key actors toward diminishing this tension, finding common ground, and moving educators toward practices that are more beneficial for students.

What Is the Source of This Tension?

How can adults who all share a common goal of helping children experience more success in mathematics be so at odds about what practices work and which practices should be avoided? In 1986, Hiebert and LeFevre wrote a chapter and distinguished conceptual understanding from procedural understanding. It is important to recognize the context in which they wrote this chapter. At the time, algorithm-only instruction was the rule, not the exception, in classrooms. We believe Hiebert and LeFevre were deliberately challenging the field of math education to aspire for teachers to understand the underlying coherent structure of mathematics, so that they could assist students to attain more substantial and lasting mathematical proficiency. In their chapter, Hiebert and LeFevre defined procedural knowledge as superficial and sequential (as opposed to rich) syntax, steps, conventions, and rules for manipulating symbols; reducing the definition to basically memorization of the algorithm only.

By 2001, the Adding It Up report asserted that pitting conceptual against procedural understanding created a “false dichotomy” (p. 122, NRC, 2001) that ultimately detracted from the goal of helping U.S. students attain greater mathematical proficiency. The NMAP (2008) report reiterated that conceptual understanding and procedural fluency are mutually beneficial and equally important. Yet, by this time, math teachers were hearing the message that conceptual understanding had to precede procedural skill building and that directly teaching algorithms could be harmful to students even from reputable sources like the National Council for Teachers of Mathematics (e.g., https://www.nctm.org/Publications/Teaching-Children-Mathematics/Blog/Strategies-Are-Not-Algorithms, published online in 2016).

In 2005, Star challenged the dichotimization of procedural and conceptual knowledge, asserting that “depth” is a dimension that can be applied logically to both procedural and conceptual knowledge. He further argued that Hiebert and LeFevre's (1986) treatment of procedural versus conceptual understanding missed the notion of heuristics (i.e., tactics used to solve problems) and underplayed the hugely important role of flexibility in mathematical problem solving. Star (2005) defined procedural knowledge as:

… order of steps, the goals and subgoals of steps, the environment or type of situation in which the procedure is used, the constraints imposed upon the procedure by the environment or situation, and any heuristics or common sense knowledge that are inherent in the environment or situation. (p. 409)
Star (2005) argued that proficient learners could use heuristics and demonstrate flexibility in choosing which procedures to use to solve problems specific to a given context. These parameters of proficiency had been ignored in Hiebert and LeFevre's original treatment.

Flexibility is a recognized element of proficiency in mathematics and it is one that cannot occur without what Star (2005) defined as deep procedural knowledge. One example of such flexibility that proficient problem solvers use is choosing the format of a proportion given the type of problem they are trying to solve (e.g., using .58 instead of 33/57, or 3/5 instead of 60%). In math, flexibility is a life skill that allows problem solvers to turn challenging problems into easier problems in the context in which a problem must be solved (e.g., what level of precision is required? What operation or operations will I need to conduct?). For example, when solving a problem 6 × (14 ÷ 6) + 10, flexible problem solvers will recognize immediately that the factor of 6 multiplied by (14 ÷ 6) is the same as 14. Problem solvers with only superficial procedural knowledge may simply apply the PEMDAS rule (parentheses, exponents, multiplication, division, addition, subtraction) for order of operations and solve 14 ÷ 6 and then try to multiply that quantity by 6, which will result in a decimal quantity due to rounding the irrational value that results when 14 is divided by 6. A flexible problem solver will simply choose to represent 14 divided by 6 as a fraction quantity (i.e., 14/6) and then the answer is apparent: 14 + 10 or 24. Mathematical problem solving is rife with these types of examples, beginning with very simple skills (e.g., finding a near easy fact in addition to solve a more challenging addition fact) and continuing through more advanced skills like solving linear equations. Given 2 (x + 1) + 3 (x + 1) = 10, a flexible problem solver will recognize that there are two ways to proceed: collect then distribute 5 (x + 1) = 10 or distribute then collect 2x + 2 + 3x + 3=10 and can choose the method that seems easiest given the problem context. However, given 2 (x + 1) + 3 (x + 2) = 10, the flexible problem solver will recognize that one must distribute and then collect to solve because the terms ((x + 1) and (x + 2)) are different terms. Flexibility requires understanding the relationships between operations, facility with creating equivalent quantities, constructing problems to solve for an unknown, and choosing the problem-solving step or steps that are easiest in a given problem-solving context.

Some teachers may be unaware of the National Council of Teachers of Mathematics (NCTM) position statement on procedural fluency that can be found at https://www.nctm.org/Standards-and-Positions/Position-Statements/Procedural-Fluency-in-Mathematics, which is consistent with Star (2005). But this very position is seemingly at odds with other recommendations offered by the same organization which is so influential among math teachers. For example, a recent president of NCTM addressed fluency with advice that can be found here (https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Linda-M_-Gojak/Fluency_-Simply-Fast-and-Accurate_-I-Think-Not!/). On the surface, there is nothing objectionable in this advice. In fact, the advice is very seductive because it seems so reasonable. However, the recommended approach for building fluency is terribly misleading because it is wholly disconnected from the empirical body of work around how to build fluency and, in fact, is at odds with best practices. A savvy school psychologist needs to know about these disconnects and find ways to hybridize classroom practices to ensure that children develop fluent performances that are built upon and in turn benefit conceptual understanding and reflect generalizable and flexible problem-solving skills. The key take-away for school psychologists is to understand that (a) procedural fluency and conceptual understanding emerge in concert around specific and connected skills, (b) high-quality fluency building instruction requires high doses of opportunities to respond (e.g., practice with feedback) that ensures high student engagement and occurs in frequent doses, and (c) knowing whether students have attained fluency requires the use of brief timed assessments (Burns, Riley-Tilman, & VanDerHeyden, 2012). It is also important for school psychologists to know that class-wide fluency-building intervention can be a powerful supplement to typical classroom practices in math even though teachers may be wary of these evidence-based practices (VanDerHeyden & Codding, 2015).

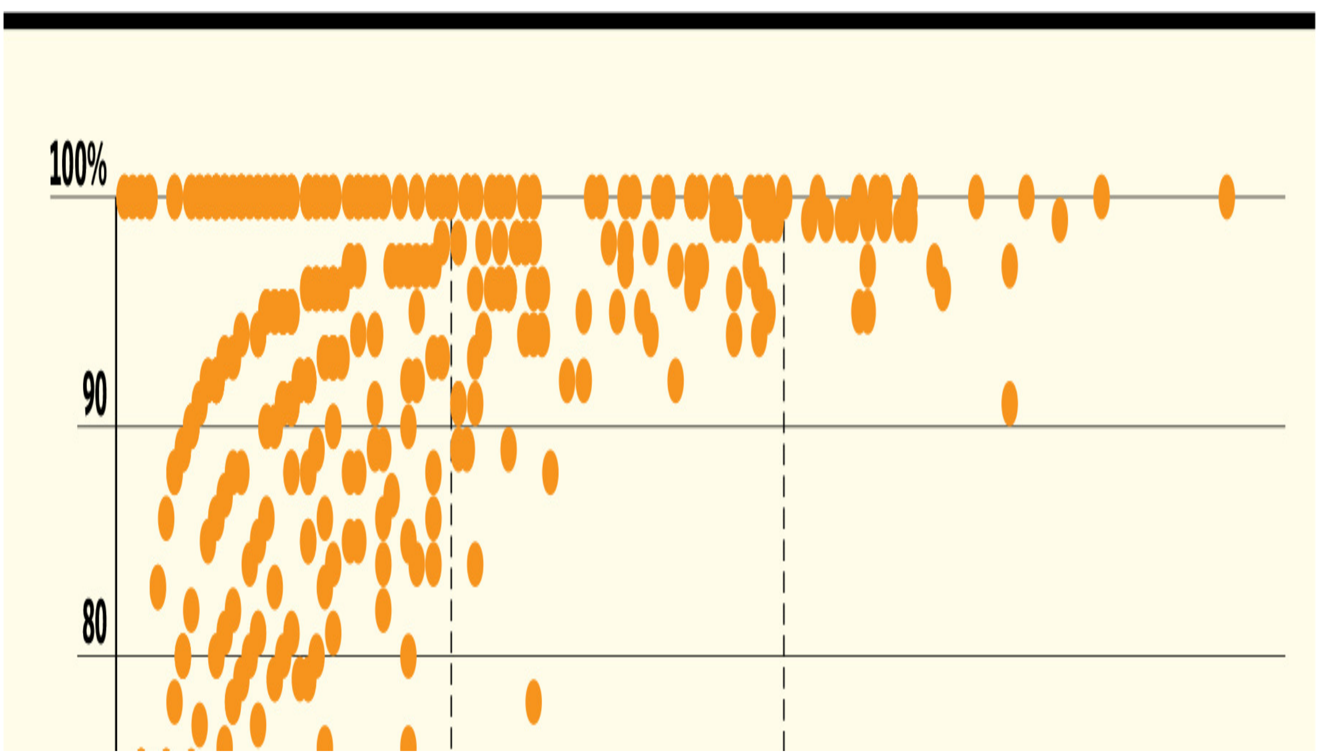
In the rest of this paper, we will discuss some common misunderstandings of math practices and summarize available evidence that school psychologists may use to advise systems in the thoughtful implementation of evidence-based practices in ways that bring mathematical success to more children, which is sorely needed in the United States.

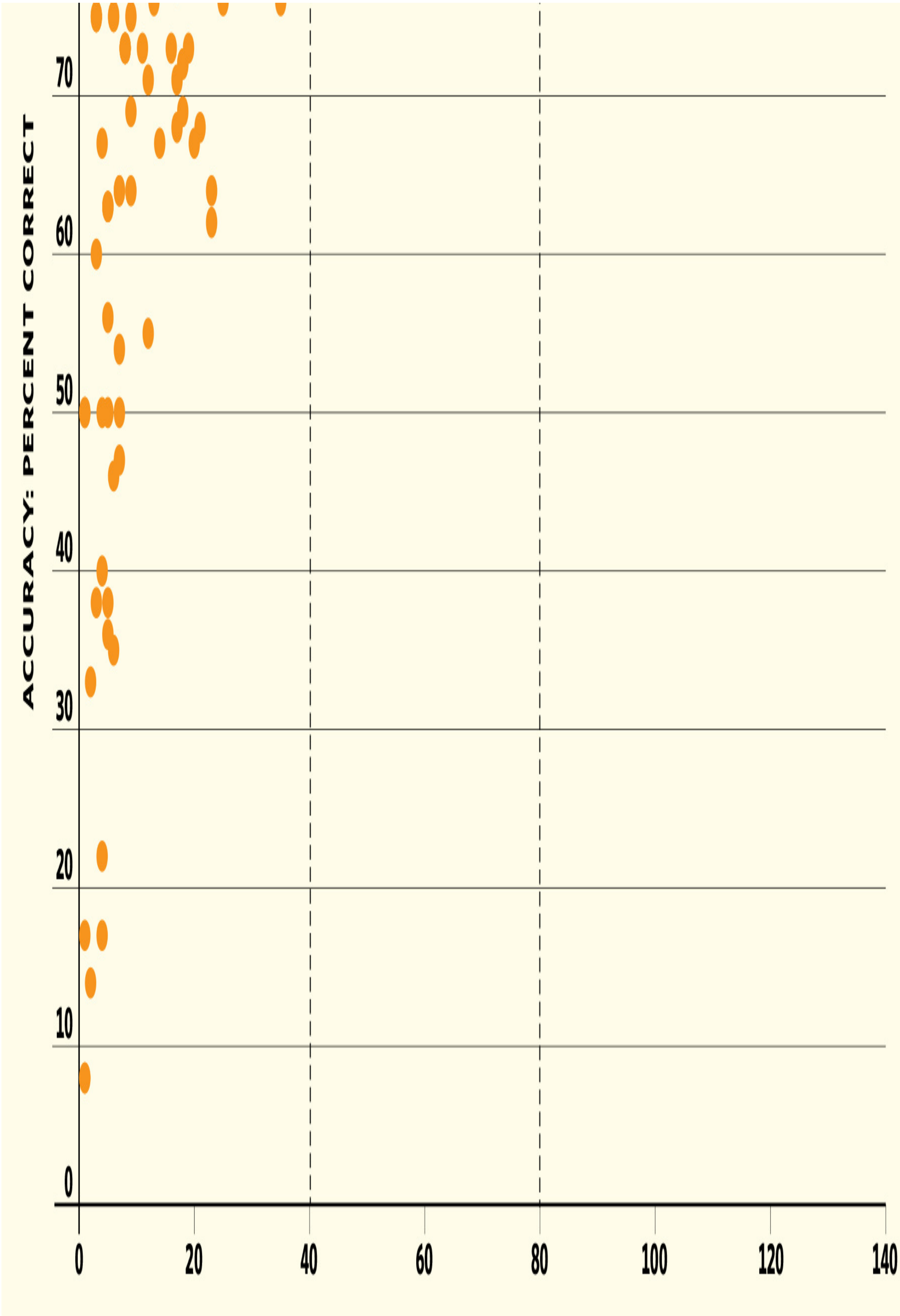
Why Timed Assessment Is Important

Teachers may view timed assessment and practice as tantamount to rote memorization, but the evidence makes a case that timed assessment is an important component of instructional decision making and that timed practice is a necessary active ingredient of fluency-building intervention. Why do we rely on timed assessment in mathematics? First, it is important to use timed assessment at certain decision points because timed assessment provides superior information than does untimed performance in terms of knowing whether students have attained mastery and whether they are ready for more challenging content. If timed assessment were not necessary to make meaningful instructional decisions, then perhaps it could be avoided altogether. So first, let's understand why we must have timed assessment.

As part of a randomized control trial of class-wide math intervention (VanDerHeyden, McLaughlin, Algina, & Snyder, 2012), fourth and fifth grade students participated in math screening in the fall (*N*=209) and the spring (*N*=218). Procedural controls were followed to ensure administration fidelity and interscorer agreement. Each measure was scored for digits correct per 2 minutes for the original study. Raw data were rescored for digits correct per minute and accuracy of responses (computed as the number of correct digits divided by the total number of digits attempted and multiplied by 100%). The data provided for this example come from the fact families measure for multiplication and division facts with numerals 0–12 administered at both time points. In the scatterplot depicted in Figure 1, for each of 427 administered and scored measures, the digits correct score is plotted against the accuracy score.

Figure 1. Fluency by Accuracy




ACCURACY: PERCENT CORRECT
70
60
50
40
30
20
10
0
0
20
40
60
80
100
120
140

FLUENCY: DIGITS CORRECT PER 2 MINUTES

One pattern that readers should notice right away is the natural tendency of errors to diminish as performance becomes more fluent. This pattern should resonate with readers, because with oral reading fluency, students making the most errors while reading are typically those students reading at lower rates. It is a natural pattern of behavior for errors to diminish as speeded performance improves.

The level of accuracy that teachers typically require to consider students proficient might be 90%. That accuracy criterion is reflected by the horizontal line connecting to the y-axis at 90%. The score in digits correct per 2 minutes (fluency score, which is accuracy plus speed; Binder, 1996) has two criteria shown as vertical, dashed bars. The one closest to the y-axis represents the instructional range and the one farther to the right represents the mastery range of performance (Deno & Mirkin, 1977). Functionally, we know that children who attain the instructional range of performance in digits correct per 2 minutes are likely to be making fewer errors (i.e., they have acquired the skill) and visually we can see that is true in this graph. Students in this stage of learning will benefit the most (i.e., grow the most rapidly) given instructional tactics that are designed to build fluency (e.g., increasing opportunities to respond, removing prompts and cues, setting goals, providing rewards, and encouraging self-monitoring of performance gains, and delayed error correction). Students to the right of the mastery line are students who are ready for generalization opportunities and more challenging problem types. We also know that students who are to the left of the mastery line and especially to the left of the instructional line are highly unlikely to retain the skill in only a few weeks, are highly likely to make errors that compromise understanding, are unlikely to be able to use the skill to solve more complex or novel problems, and will not experience faster learning of more complex related content (Burns, VanDerHeyden, & Jiban, 2006). So here is the punchline: When a teacher uses untimed assessment to judge whether students have mastered important mathematical content and understandings, the teacher will be wrong in all those cases that fall above the 90% criterion but to the left of the instructional line. This is a large number of cases (*n*=157 errors or 55% of cases in the frustrational range) about whom the teacher would reach an incorrect conclusion and deprive students of additional needed instruction to truly attain mastery.

This problem is one that is expected based on the accuracy metric. Once a student reaches 100% accuracy, there is nothing more that the accuracy metric can tell you about proficiency and yet, there is valuable information still to know. If you have two students who both score 100% on an addition task, but one student has to draw and count hashmarks while the other student can solve problems immediately or employ a variety of efficient strategies to arrive at the correct answer (e.g., 5 + 4 = 5 + 5 -1), the second student is more proficient and the only way to detect that superior proficiency would be to time the performance. Under timed conditions, the second student would answer more problems correctly than would the first. This truth of assessment (and the limits of accuracy on untimed measures) is exactly why college readiness batteries use timed assessment. It is the timing that separates the 30s from the 36s on the ACT, for example.

Tension 1: Teachers Might Believe That Math Instruction and Assessment Cause Anxiety

Teachers and parents worry about math anxiety, and some math education experts caution against tactics used in math class, such as timed tasks and tests, that might theoretically stoke anxiety (Boaler, 2012). First, the evidence does not support that people are naturally anxious or not anxious in the context of math assessment and instruction (Hart & Ganley, 2019). Second, simply avoiding math or certain math tactics should not be expected to ameliorate anxiety in the long term. Third, preventing a student from full exposure to math assessment and intervention costs the student the opportunity to develop adaptive coping mechanisms to deal with possible anxiety in the face of challenging academic content. Fourth, focusing solely on math anxiety overlooks the important role that schools and teachers can play in reducing anxiety so children can participate fully in math instruction.

Given this important rationale for timed assessment, what do we know about anxiety in math? Gunderson, Park, Maloney, Bellock, and Levine (2018) found a reciprocal relationship between skill proficiency and anxiety, such that weak skill reliably preceded anxiety and anxiety further contributed to weak skill development. They found that anxiety could be attenuated by two strategies: improving skill proficiency (this cannot be done by avoiding challenging math work and timed assessment) and promoting a growth mindset (as opposed to a fixed ability mindset) using specific language and instructional arrangements to promote the idea that I, as a student, can work hard and beat my score; I can grow today; my brain is like a muscle that gets stronger when I work it with challenging math content. A recent meta-analysis that included 131 studies also found a negative correlation between anxiety and math performance (r=-.34) and the negative relationship between math anxiety and math performance was stronger when performance was measured on complex, multistep math tasks and when students believed that the math task would impact their grades (Namkung, Peng, & Lin, 2019).

Research by Hart and Ganley (2019) found the same association between math skill and adult math anxiety. Most adults in their study with about 1,000 participants reported low to moderate math anxiety. Self-reported adult math anxiety was negatively correlated with fluent addition, subtraction, multiplication, and division performance (r = -.25 to -.27) and probability knowledge (r = -.31 to -.34). Self-reported test taking anxiety was negatively correlated with math skill fluency and probability knowledge, too (r=-.22 to -.26). One must wonder with these emerging data whether math anxiety has been oversimplified in the press.

What does the evidence say about math anxiety? There is very little empirical evidence examining whether timed tests have a causal impact on anxiety, and the existing few studies that include school-age participants do not support the idea (Grays, Rhymer, & Swartzmiller, 2017; Tsui & Mazzocco, 2006). What is clear is there is a modest, negative bidirectional relationship between math anxiety and math performance (Namkung et al., 2019). These correlational data suggest that poor mathematics performance can lead to high math anxiety and that high math anxiety can lead to poor mathematics performance. The remedy that school psychologists can advocate for is to identify, through effective and efficient screening, the presence of high math anxiety and determine which students would benefit from supplemental and targeted mathematics supports. Intervention approaches should target math skill deficits, address high anxiety, and promote a growth mindset as well as monitor progress toward clearly defined objectives using tools that are brief (often timed), reliable, and valid.

A substantial body of longitudinal and experimental research has emerged examining exactly this question (see Rittle-Johnson, 2017 for a review). The empirical evidence has demonstrated that the purported unidirectional relationship between conceptual understanding and procedural knowledge is untrue. For example, Hecht and Vagi (2010) found that procedural knowledge with fractions among fourth graders predicted their conceptual knowledge with fractions as fifth graders and vice versa even after

Tension 2: Teachers Might Believe That Instruction in Algorithms Is Harmful and Conceptual Understanding Must Precede Procedural Knowledge

controlling for prior knowledge. Schneider, Star, and Rittle-Johnson (2011) also demonstrated that procedural knowledge predicted conceptual knowledge and vice versa across a broad array of skills and concepts. Knowledge development is iterative and understanding, which can be thought of as more conceptual than procedural, facilitates procedural knowledge and procedural knowledge facilitates deeper conceptual understanding. Effective instruction includes both and not in a linear fashion, but in a way that facilitates bidirectional input and opportunity and results in understanding and performance that is flexible, retained, adaptable, and useful in learning new, more complex content.

By the time of the Adding It Up report in 2001, the research was clear that avoiding algorithm-only instruction was and is distinct from not teaching the algorithm at all. Despite this evidence and clear recommendations from Common Core State Standards in Mathematics (2010) and NMAP (2008) indicating that children should master the standard algorithm, such instruction currently takes a back seat to alternative approaches to problem solving. The standard algorithm serves as a link between conceptual understanding and procedural knowledge. Without understanding the logic of why the standard algorithm works, students will be unable to determine when to appropriately apply the standard algorithm (Fuson & Beckmann, 2012–2013; Wu, 2011). However, given that conceptual understanding and procedural knowledge are bidirectional, students may better understand the conceptual basis for the standard algorithm by applying the procedure. Standard algorithms work because of mathematical laws; they can, and should, be proofed and unpacked. Standard algorithms require students to decompose numbers into base-10 units and complete a series of simple computations. They are systematic, efficient, and transferrable. For example, if students understand how to use the standard algorithm to solve 2-digit by 2-digit operations, they can generalize this approach to larger whole number problems as well as operations with decimals (Fuson & Beckmann, 2012–2013). The standard algorithm is one of several key approaches that students need to be explicitly taught in order to engage in more flexible and efficient problem solving.

What does the evidence say about procedural and conceptual knowledge acquisition? In order to solve problems flexibly and efficiently, students need to be exposed to instruction that is both conceptual and procedural (NMAP, 2008; Schneider et al., 2011). The empirical data indicate that conceptual understanding and procedural knowledge are bidirectional (e.g., Rittle-Johnson et al., 2015). Therefore, school psychologists can advocate for core instructional approaches and adoption of curriculum that interleave conceptual and procedural lessons. Additionally, school psychologists can assist teachers in recognizing that the standard algorithm is an important tool to explicitly teach in order to illustrate to students the relationship between concepts and procedures.

Tension 3: Teachers Might Believe That Explicit Instruction Is Beneficial Only for Struggling Learners

The most robust finding demonstrated across multiple meta-analyses is that explicit instruction is the most effective mathematics instructional practice (e.g., Gersten, Chard, et al., 2009; Hattie, 2009; Swanson, 2009). In fact, the strongest effects reported in experimental research on mathematics achievement are for explicit instruction (*d*=.55; Hattie, 2009). It has also been demonstrated that students with mathematics difficulties, as well as students with other types of disabilities, benefit more from explicit instruction than discovery-oriented approaches (Kroesbergen & Van Luit, 2003). The finding that explicit instruction is essential for students with mathematics disabilities or difficulties is plainly evident in the literature and led to recommendations from a panel within the Institute for Education Sciences that tiered intervention supports embed explicit instruction (Gersten, Beckmann, et al., 2009). The importance of explicit instruction for typically performing students has been less often studied (Doabler et al., 2015). Although the existing evidence has been mixed, the NMAP (2008) report indicated that the current data warrant the inclusion of explicit instruction along with student-centered approaches in core instruction. A recent study demonstrated that the rate and quality of student–teacher interactions, embedded within an explicit instruction approach to core kindergarten instruction, was related to student achievement (Doabler et al., 2015). Collectively, these data suggest that explicit instruction should be incorporated in universal teaching practices, and if it is not, access to core mathematics instruction is going to be limited for many students.

Because of the ubiquitous use of the phrase, confusion about the precise definition of explicit instruction is common among educators. Sometimes explicit instruction is confused with Direct Instruction (Stein, Kinder, Rolf, Silbert, & Carnine, 2018), which is a specific, highly effective, instructional curriculum that employs explicit instruction. Explicit instruction may also be confused with didactic lectures. Explicit instruction is a systematic approach that incorporates previewing of previous skills and concepts, precise instructions, modeling, guided and independent practice, immediate feedback, and checks for maintenance of skills. This methodology is aligned with student proficiency (acquisition, fluency, generalization/application) as support is scaffolded for frequent student responding. Explicit instruction provides highly engaging learning volleys between content, teachers, and students that build many opportunities for practice, verbalization, feedback, and demonstration of mathematical thinking, which in turn, sets the stage for successful skill acquisition and mastery, and enables creative expression and curious exploration. Teachers can easily differentiate instruction to provide enrichment for advanced content or address foundational skill deficits that are impeding grade-level performance. Step-by-step routines incorporate opportunities for children to respond in multiple formats (verbally, drawing, constructing, and writing). Explicit instruction anticipates, prevents, and detects misunderstandings with carefully engineered lesson content. Explicit instruction draws direct connections between what a student already knows and what a student is currently learning. As a result, the use of explicit instruction provides opportunities to reason, speculate, estimate, justify, predict, conclude, and ask new questions. In sum, explicit instruction is a cornerstone of effective mathematics instruction (VanDerHeyden & Alsopp, 2014).

What does the evidence say about the value of explicit instruction in math? The scientific evidence on the benefits of explicit instruction is clear and robust, suggesting that school psychologists should feel comfortable supporting teachers in the use of explicit instruction in their general education classrooms during core instruction. School psychologists should also advocate for the adoption of tiered intervention supports that incorporate explicit instruction. The IRIS Center (https://iris.peabody.vanderbilt.edu/module/math) and National Center for Intensive Intervention (NCII; https://intensiveintervention.org/intensive-intervention-features-explicit-instruction) have online modules describing explicit mathematics instruction to which school psychologists can direct administrators and educators . Notably, systematic review of different mathematics textbooks across grades 1, 2, and 4 all identified the need for more explicit instruction to be embedded, among other critical evidence-based instructional principles (Doabler, Fien, Nelson-Walker, & Baker, 2012; Sood & Jitendra, 2007). Therefore, school psychologists should also participate in school-level discussions regarding the adoption of curriculum and be sure that adequate opportunities for explicit instruction are embedded. If school psychologists work in schools where explicit instruction is not embedded into the curriculum or instructional routines, then consultation with teachers can be used to develop activities that embed explicit instruction to supplement core instruction.

Tension 4: Teachers Might Believe That Executive Functioning Tools and Interventions Improve Math Performance

The relevance of cognitive measures for intervention planning has long been debated despite consistent evidence indicating that cognitive measures are not helpful for intervention planning or associated with intervention outcomes (Miciak, Williams, Taylor, Cirino, Fletcher, & Vaughn, 2016; Stuebing et al., 2015). The aptitude by treatment interaction theory suggested that instructional interventions are more or less effective depending upon students' measured cognitive aptitudes. Although this theory was refuted by Cronbach and Snow in a meta-analysis published in 1977, the notion that matching cognitive aptitudes, and more recently executive functions, to instructional interventions has persisted. The evidence summarized and analyzed in meta-analytic studies illustrates that (a) although cognitive measures correlate with mathematics achievement, these measures do not correlate with student responsiveness to intervention; (b) using cognitive assessment tools does not provide the information necessary to improve academic skill weaknesses; and (c) cognitive interventions do very little to improve academic performance outcomes (Burns, 2016). In their meta-analysis, Jacob and Parkinson (2015) found a moderate association, that remains consistent across developmental levels, between executive function skills and math and reading achievement. Importantly, the authors noted that most studies failed to control for IQ or background characteristics and when studies did control for these factors, the strength of the relationship between executive functioning and achievement was reduced. This meta-analysis also examined intervention studies that employed executive function interventions and measured outcomes on both executive functioning and academic achievement. These authors concluded that there are very few rigorous intervention studies examining the causal link between executive function interventions and academic outcomes. The authors then indicated that these existing studies showed improvements on measures of executive function but no improvements on academic achievement. Thus, the notion that executive function training can bring about gains in mathematics proficiency is not consistent with existing evidence.

The evidence serves as a reminder that the most effective way to address a math skill deficit is to directly remediate math skills rather than trying to improve working memory or executive functioning as a means to address math skill deficits. This conversation is not intended to suggest that individual differences do not matter. In fact, there is some preliminary data suggesting differences in working memory and reasoning impact intervention responsiveness (Fuchs et al. 2013; Fuchs et al., 2014). For example, Fuchs and colleagues (2013) compared a number knowledge intervention that either included a fluency-building activity or a conceptual-knowledge activity delivered to first grade students at-risk for mathematical difficulties. The findings suggested whether students had weak or strong cognitive reasoning ability did not matter when they received the intervention with the fluency activity, but for students with low reasoning ability, the intervention with the conceptual activity led to poorer outcomes than their peers with better reasoning ability. Similarly, Fuchs and colleagues (2014) evaluated a fraction intervention provided to at-risk fourth graders that either embedded fluency or conceptual practice activities. The findings suggested better performance in either intervention group compared to the control. However, students with very poor scores on a working memory task did better with the intervention variation that included a conceptual activity whereas children with more adequate scores (all students had relatively low working memory scores) did better on the intervention variation including the fluency activity. It is important to recognize in both studies the way the specific math activities were constructed within the academic intervention was altered to address individual cognitive differences; therefore, the emphasis was still on the math skill itself.

What does the evidence say about executive functioning and math performance? The evidence supports providing intensified instruction for students who struggle in the context of general education mathematics instruction. Interventions should be tailored and intensified according to student needs using direct evaluation of students' skills to make low-inference decisions about intervention tactics. It has long been recognized that students at-risk for mathematics learning disabilities may also have difficulties with attention, motivation, self-regulation, and working memory (e.g., Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2012; Montague, 2007). Thus, when building intensive interventions, it is useful to include self-regulation and reinforcement strategies, minimize cognitive load on working memory and reasoning by including explicit instruction and breaking down problems into smaller more manageable parts, minimize excessive language load by incorporating visual representations, and provide fluency practice (Fuchs, Fuchs, & Malone, 2018; Powell & Fuchs, 2015). School psychologists can help establish clear, systematic guidelines within school intervention teams for adapting interventions and intensifying instruction in order to address students with the largest gaps between expected and present performance.

Conclusion

In conclusion, we summarize incorrect beliefs that create tension between those beliefs and evidence-based practices. We attempt to contextualize these incorrect beliefs to create common ground, and summarize contemporary evidence (see Table 1). School psychologists are, first and foremost, advocates for all students, which necessitates a strong commitment to the use of practices that can be expected to work if correctly implemented (i.e., evidence-based practices). But school psychologists must be highly savvy implementation allies to support the adoption of effective practices in schools. In math, many children are vulnerable to the use of ineffective practices because teachers and teacher leaders have been influenced by modern math myths. The school psychologist can play a pivotal and much needed role in inspiring, equipping, and reinforcing the use of effective practices for students to help more students realize the lifetime social and economic benefit of math proficiency.

Table 1. Commonly Held Beliefs, Their Origins, and Evidence-Based Positions

Skill is precursor to anxiety. Growth mindset practices reduce anxiety and increase skill.Procedural and conceptual knowledge are bidirectional so conceptual knowledge is worsened.

Incorrect Belief	**Where does this incorrect belief come from and What Harm May it Cause?**	**What is the Empirically Supported Position?**
Timed assessment causes anxiety.	Fear that timed assessment will cause anxiety that will be harmful for the child and justifies avoidance of timed assessment (i.e., risks outweigh benefits belief). When teachers avoid timed assessment, they lose the ability to accurately estimate skill mastery, which means they cannot bring formative assessment to bear on learning.	
You cannot teach a child how to procedurally solve a problem until you have established conceptual understanding.	An incorrect belief that teaching procedural knowledge will be a barrier to conceptual knowledge and that understanding progresses linearly. Failure to teach problem solving procedures and strategies actually inhibits conceptual understanding.	Relationship between procedural knowledge and conceptual knowledge is bidirectional.
It is harmful to explicitly teach a child how to use an algorithm to solve a problem.		Algorithms are like phonics skills in reading. They work because of mathematical laws. They can be proofed and unpacked.
Explicit instruction is beneficial only for struggling learners.	Lack of understanding of what explicit instruction actually entails. Many believe that explicit or direct instruction involves a teacher providing didactic lectures to students. Some teachers fear that using explicit instruction will harm learners and so they avoid.	Over 50 years, explicit or direct instruction has yielded the strongest stable effect size on learning for students at all performance levels.
Executive functioning tools and interventions improve academic performance.	Belief that executive function skills, which are correlated with academic proficiency, are causal rather than just correlated with academic proficiency. Unnecessary additional assessments are administered instead of making low inference instructional decisions based upon direct evaluation of skill strengths and weaknesses leading to the recommendation of intervention tactics that will not produce the desired academic gains.	Executive functioning (and cognitive) interventions do not improve academic outcomes.

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