

# Making Math Meaningful

Supporting Students  
With Autism in Math

## Presenter: Randy Ewart



- 20 year math teacher
- Master's in special ed
- Adjunct USJ – sped math
- Math interventionist
- Candidate for National Board Certification in sped

Gabriel Ewart  
9 year old with autistic disorder

## Objectives

- Differentiate between a concept and a skills
- Make math topics meaningful

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### Initiation

**Part 1: Task Analysis for Math Topics:**

...strategies for identifying valid learning objectives, breaking them into small parts, and then using task analysis as an assessment tool.

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### Task Analysis

- Task Analysis is a formal procedure for breaking the topic into manageable little parts for the students
- It can be used to guide assessment:
  - For all steps
  - For subset of steps

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### Given Objective

Objective: Compute total to pay given tax rate and price of multiple items to be purchased.  
(solve multi-step real life problem)

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### Used Task Analysis to Identify Gaps

Steps		Outcome C = correct, 0 = prompt, 1 =
1	Discern difference between money amounts and tax rate	
2	Find total cost , before tax	
3	Write total with proper notation	
4	Discern difference between tax rate and tax as money amount	
5	compute tax	
6	Identify need to add (pay both total cost and tax)	
7	Compute total to pay	
8	Write total to pay with proper notation.	
9	Identify total to pay orally.	

### Initiation

#### Part 2: Making Math Meaningful for Students with an Autism Spectrum Disorder:

...strategies for making math topics meaningful for students

### Agenda

- Foundation for meaning
- Strategies to make math meaningful

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### Documents to Share

- [www.ctspedmathdude.com](http://www.ctspedmathdude.com)
- Categories: webinar, presentations
- Provides link to Dropbox folder containing documents

### Foundation for Meaning

### What do we mean by a concept?



- A. How to perform a math task
- B. The idea behind a math topic
- C. A fact about a math topic
- D. None of the above

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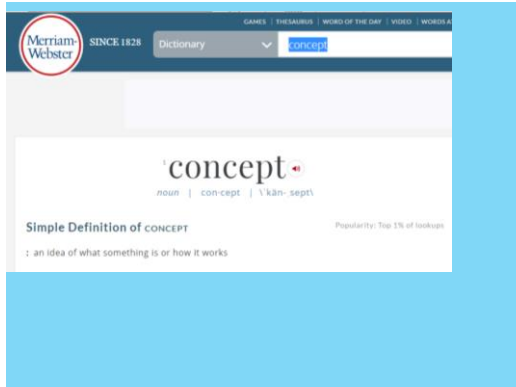
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## Concept

Collection of facts about a topic that results in an idea that is greater than the sum of the individual facts.

The underlying idea of what a topic is as opposed to how to perform steps to "do" the topic.

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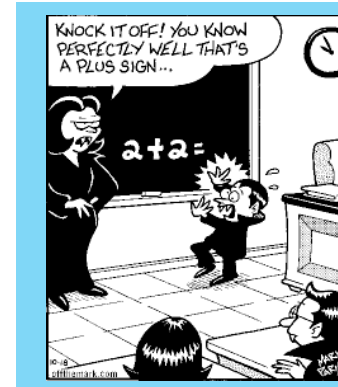
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Math is a language with its own symbols

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You have 90 seconds to memorize the following words. Do not write any of the words as you memorize them.

- Bill carrot legos cat train duck John celery puzzle boat pig Mary car spinach ball

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**Which strategy would you use to memorize the words?**



- A. Rehearsal – rote memorization
- B. Make up a story
- C. Focus on categories
- D. Create visuals for each word

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### **Gestalt Theory**

- Brain wants to make meaning out of information so most people choose B, C or D.
- The brain wants to see the big picture – see the forest among the trees.

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What do you see in each image?

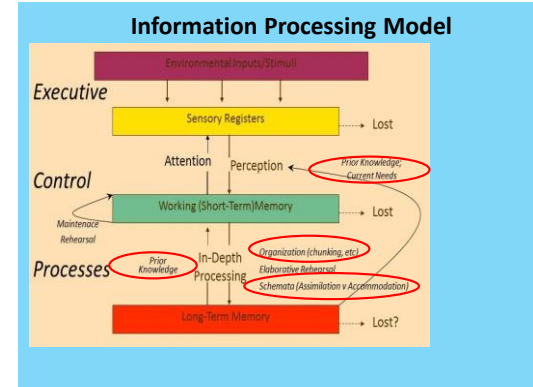
Panel A: Three black semi-circles arranged in a triangular pattern, each with a white notch pointing towards the center.

Panel B: A black shape resembling a stylized letter 'S' or a curved line.

Panel C: A white starburst shape with multiple sharp points radiating from a central point.

Panel D: A black shape resembling a stylized letter 'C' or a curved line.

What do you see?




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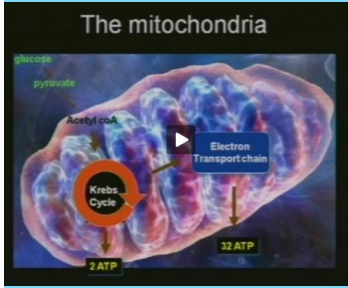
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### Explain Mitochondria



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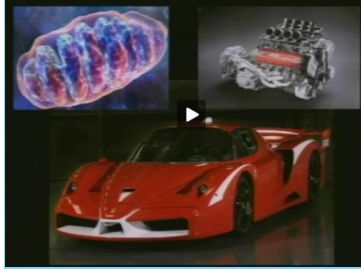
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### The Mitochondria is the Motor for the Cell



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### Strategies for Making Math Meaningful

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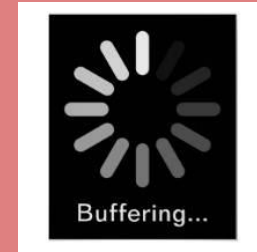


### Strategies for Making Math Meaningful

- Highlighting parts
- Making topics concrete
  - Relevance
  - Manipulatives (hands on)

### Highlighting Parts

### Cut Down on Mental Tasks



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### 8th Grader's Assessment Practice

6.15 The table below shows the total Connecticut government full-time employees for three years.

Number of Employees	Year
1999	528,962
2000	
2001	

Explain the trend for the number of full-time employees.

The number of employees from 1999 to 2001 all together is 528,962. The number of employees of each year goes up.

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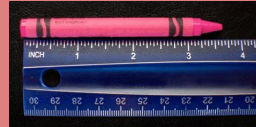
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### Identify a challenge for a student in measuring object to nearest fourth inch.



- A. Lining up the ruler properly.
- B. Identifying whole inches
- C. Identifying fourths marks
- D. All of the above




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### Identifying fourths on a ruler




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### Identifying fourths on a ruler

a response to problems identified in data sheet the student was provided focused work on identify and understanding fourths.

Fourth's Handout - Respond to each group by identifying fourths (e.g. 3 out of 4) and EXPLAIN (e.g. 3 guys wearing a hat out of 4 guys total)

How many hats are there?				
How many are there?				
How many M&M's are there?				
How				

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### Identifying fourths on a ruler

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How many are there?				
How many M&M's are there?				
How				

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### Combine Like Terms

$$7.5x + 1 - 3x - 5$$

$$7.5x + 1 - 3x - 5$$

$$7.5x - 3x + 1 - 5$$

$$4.5x - 4$$

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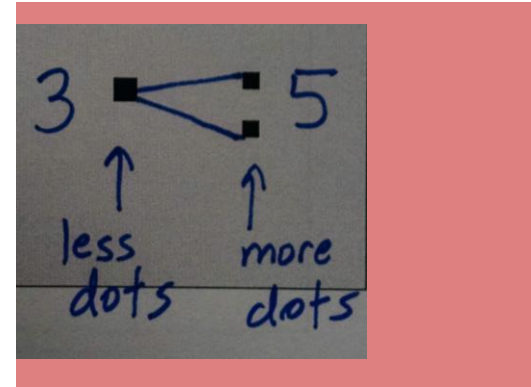
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**Identify Parts for an Inequality symbol** > <



- A. Alligator eats the bigger number
- B. Direction
- C. One side is wider than the other




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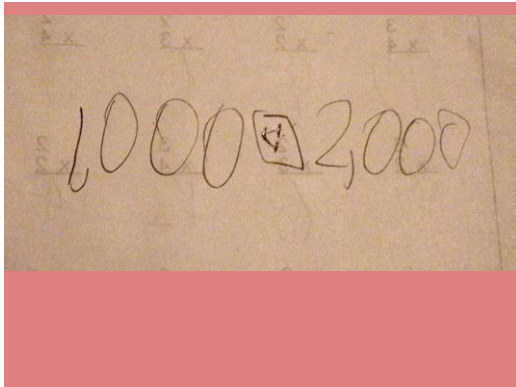
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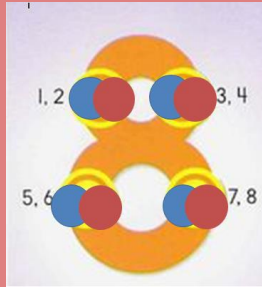
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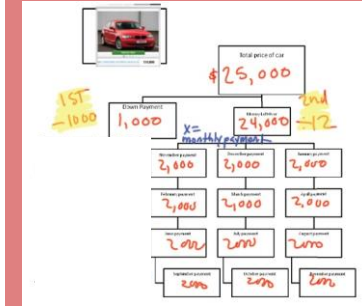
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Revising Touch Math – Colorado PD



Divide total among 12 months




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### Ordered Pairs

what point is located at each ordered pair.

1. (3)	2. (2, 1)	3. (-4, 3)
4. (7, 8)	5. (-4, 4)	6. (-6, 2)

write the ordered pair for each given point.

7. A	8. M	9. P
10. G	11. Q	12. N

plot the following points on the coordinate grid.

13. (-4, -3)	14. T (2, -4)	15. U (5, 8)
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X axis  
vs  
y axis

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### Buying a Car

The price of a used Toyota Camry as a function of miles driven is shown below.

Miles Driven	Price
0	19,600
10,000	18,600
20,000	17,600
30,000	16,600
40,000	15,600

What happens to price as mileage increases?

Price decreases

price decreases

no, it costs more

Shows a relationship

price decreases

Miles increase

$y = 19,600 - 50x$

$y = 19,600 - 50(0)$

$y = 19,600$

$y = 19,600 - 50(10)$

$y = 18,600$

Focusing on  
x variable  
and  
y variable

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### Explain what is meant by $x > 2$

your turn

- A. Alligator eats the X
- B. Draw a line to the left and an open circle on the 2
- C. All numbers bigger than 2

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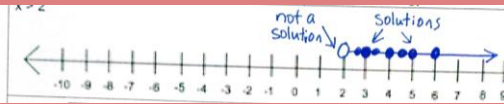
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Graphing  $x > 2$  typically is presented as drawing an open circle and a line.

Symbol	Meaning	Closed or Open Circle
$<$	Less Than	Open ○
$>$	Greater Than	Open ○
$\leq$	Less Than or Equal to	Closed ●
$\geq$	Greater Than or Equal to	Closed ●

### Parts of $x > 2$



### Relevance

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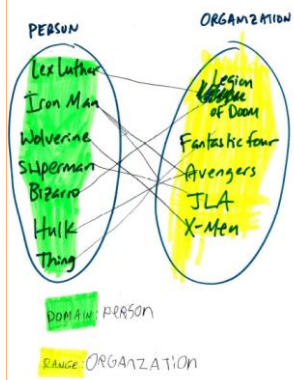
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**HS Student with autism who likes comics**

Teaching the Concept of **More**

Which one do you want? Why?



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### Counting Money



### Counting Money

- Classroom café with students ordering then counting out money to pay for food.

			
How many tacos do you have?		and	How many tacos do you eat?
3			2
3T		+	-2T
		What's left?	
		1T	

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### Rates or Slope

**Pizza Hut Pizza costs \$10 and \$2 per topping**

3. How much would it cost to get pepperoni, mushrooms and peppers? Show work.



10  
2  
2  
2  
2  
2  
2  
+  
16

4. Write an equation to show the relationship between cost and # toppings.

Cost = y    # toppings = x

$$\frac{\text{cost}}{y} = \frac{\cancel{10}}{\text{price for pizza}} + \frac{\cancel{2} \cdot \cancel{2} \cdot x}{\text{price per topping} \cdot \# \text{ toppings}}$$

### Triangles



### Word Problem for 7<sup>th</sup> Grader with Autism Who Loves Snowboarding




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**Concrete (hands on)**

**CRA**

**concrete**



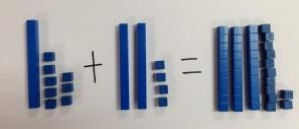
**representational**



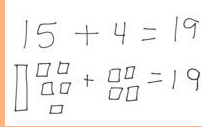
**abstract**



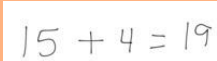
**concrete**



**representational**



**abstract**



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### 4<sup>th</sup> Grade Class – Comparing Fractions

Circle the fraction that is greater.

2/6

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
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4/6

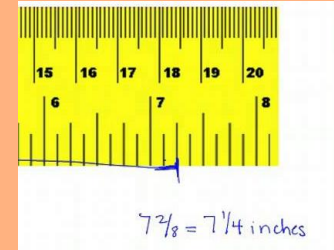
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
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### Making Temperature Meaningful

Degrees Fahrenheit	Item	Degrees Celsius	Hot Warm Cold

### Count tick marks – 2 not 3 tick marks




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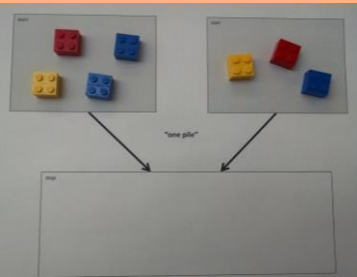
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**digging Stones - later in Fractional Disk on Number Line**

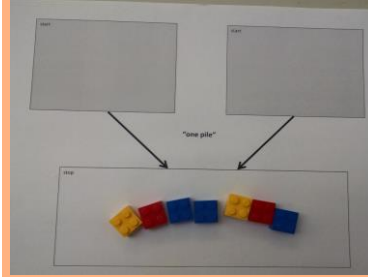
	← This is a step.
	How many steps does it take to get across?
	How many steps does it take to get across?
	How many steps does it take to get across?
	How many steps does it take to get across?
	How many steps does it take to get across?

### Concept of Adding



The diagram illustrates the concept of adding. It shows two separate boxes, each labeled "one pile". The first box contains three blocks: one red, one blue, and one yellow. The second box contains two blocks: one red and one blue. Arrows from both boxes point to a larger box below, also labeled "one pile", which contains all five blocks combined: one red, one blue, one yellow, one red, and one blue.

### Concept of Adding



The diagram illustrates the concept of adding. It shows two empty boxes, each labeled "one pile". Arrows from both boxes point to a larger box below, also labeled "one pile", which contains a row of five colored blocks: one yellow, one red, one blue, one yellow, and one blue.

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*Simplify*  $7.5x + 1 - 3x - 5$

$7.5x$	$+ 1$	$- 3x$	$- 5$
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*Simplify*  $7.5x + 1 - 3x - 5$

$7.5x$	$- 3x$	$+ 1$	$- 5$
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**Counting Money**

 3	 2	 0
 2	 0	 0
 1	 2	 2
 2	 0	 3
 0	 8	 7
How much money in all?	How much money in all?	How much money in all?

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How can we make the concept of rate, e.g. \$ per gallon, concrete?



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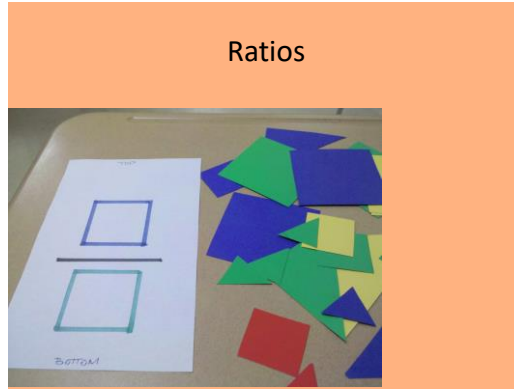
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### How can we effectively introduce the topics perimeter and area for a rectangle?



- A. Show the formula
- B. Show a rectangle
- C. Have students count sides of a rectangle

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I gave the students pieces to build a fence for their animals



$P=2L+2w$

Adding + and Multiplying \*

	+		=	
6	+	6	=	12
2 • 	=			
2 • 6	=	12		

7<sup>th</sup> grader with autism who loves cars

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## Units of Area and Volume

**Area and Volume Activities**

**Area**

- Count the number of squares in the taped off area.
- Area = 

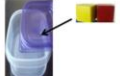
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- Measure the sides of a square using a ruler.
- Write in the unit in the squares on your paper →



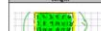
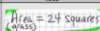

**Volume**

- Count the number of cubes that fit into the box.
- Volume = 

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**Length, area and volume units**

Length	Area	Volume
		

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Carl and Beneta are playing a game using this spinner.



Carl will win the game on his next spin if the arrow lands on a section labeled 6, 7, or 8.

Carl claims **it** is likely, but not certain, that he will win the game on his next spin.



How can we make the concept of **likely** concrete

5<sup>th</sup> grader with anxiety who loves horses



10 barns make a ranch



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### Summary

- Math is a language with its own symbols
- The symbols represent concepts
- Make the language and the symbols meaningful
- Strategies:
  - Highlight the parts of the topic
  - Make the concept relevant
  - Make the concept more concrete

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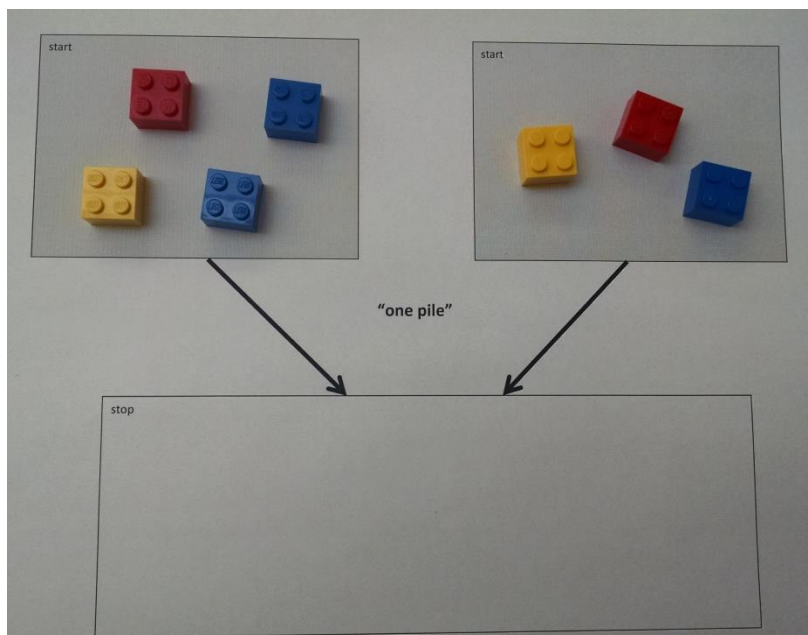
Adding numbers together involves skill but also an underlying concept of combining values/quantities to get a total. The concept is often overlooked. The following is an approach to teach and assess the concept of addition.

Overview of the steps presented on the following pages:

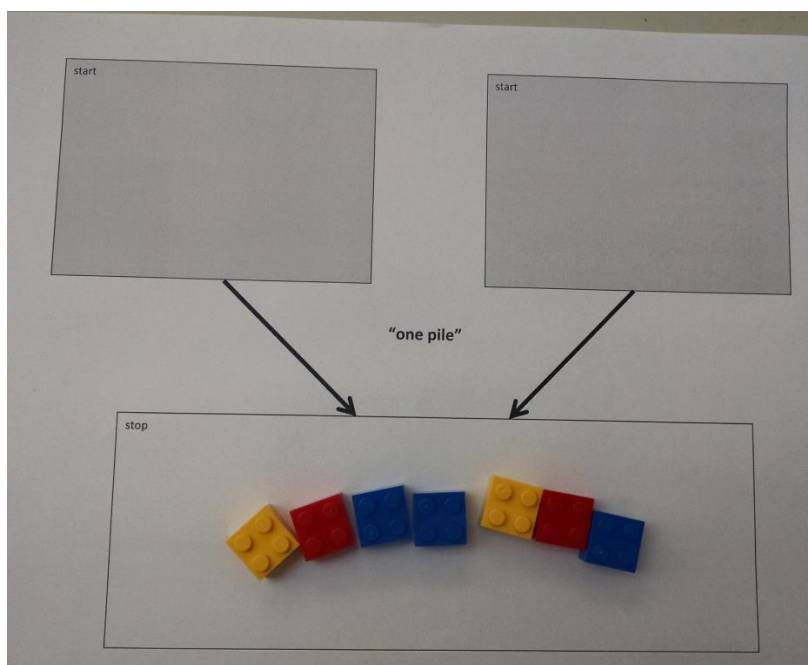
- Phase 1
  - A Concrete-Representational-Abstract (CRA) approach is used by focusing on a concrete view of addition.
  - Students are taught to physically demonstrate addition with manipulatives – the initial terms is “pull together”
  - They are also taught 2 other tasks: taking away and sorting.
- Phase 2
  - These other 2 tasks are used as distractors. The idea is that when prompted to “pull together” the student demonstrated the addition task as opposed to one of the other tasks.
- Phase 3
  - The use of the symbol “+” is gradually incorporated.

**PHASE 1****STEP 1**

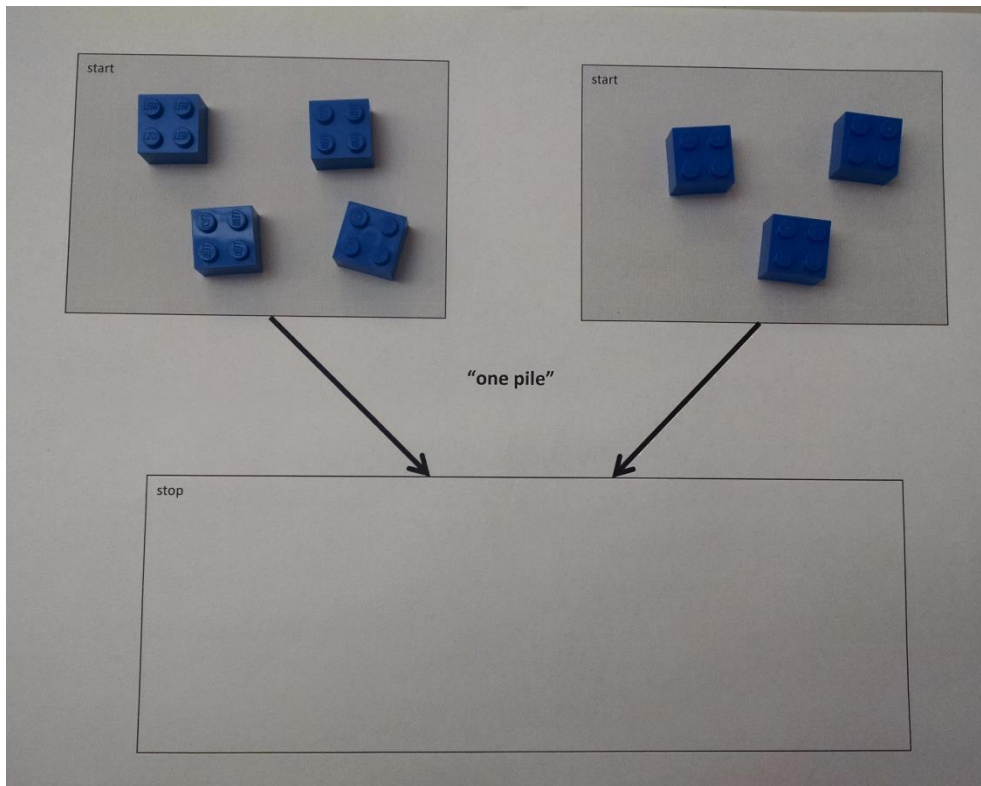
1. Use the “one pile” mat
2. Put items in both boxes.
3. Give oral prompt to “pull into one pile” or “pull together” (your choice, be consistent) – see photo below



4. The end result should look like what is shown in the photo below.
5. Ask the student to find the total. You may use “how many” and move towards the term “total.”

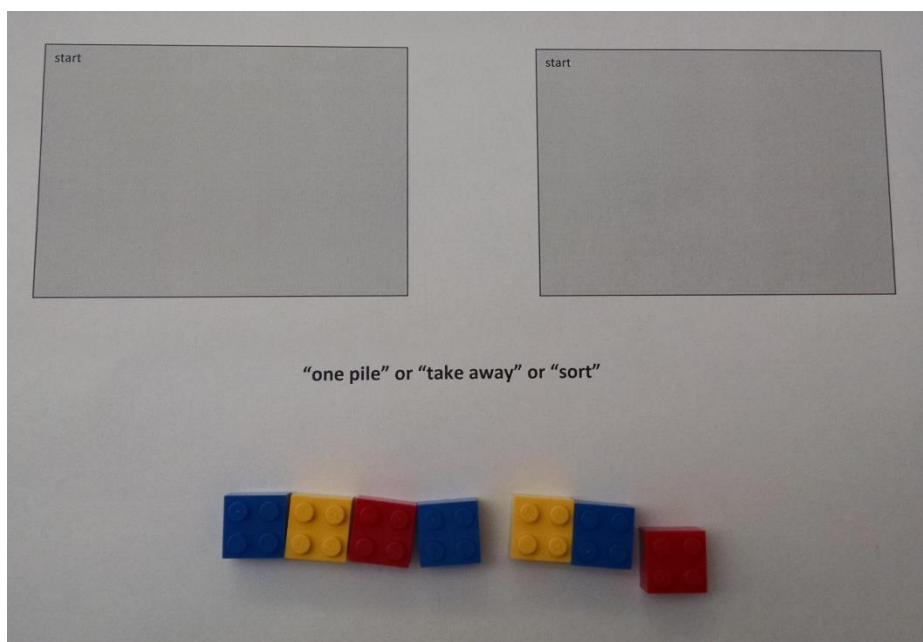
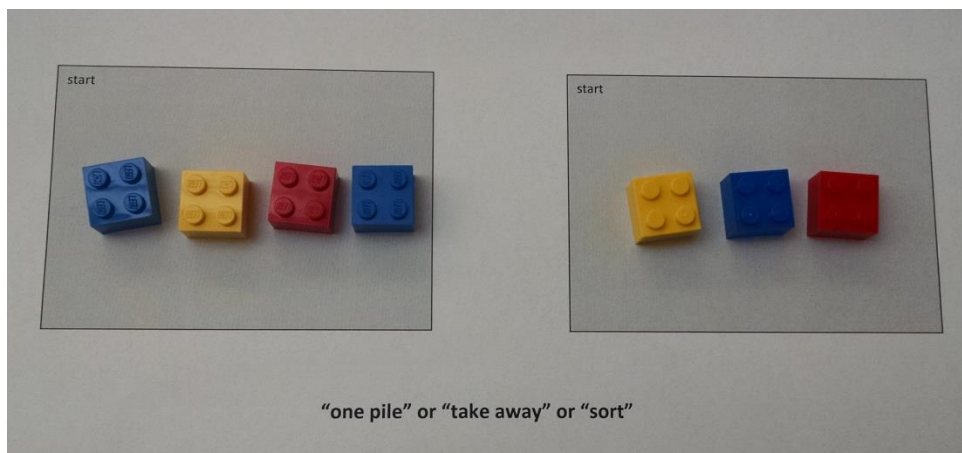


Note: If the different colors cause confusion, use a uniform color to develop the “pull into a pile” performance then move to multi-colors. The reason for using multi-colors is to allow “pull together”, “take away” and “sort” to all have the same set of Legos.



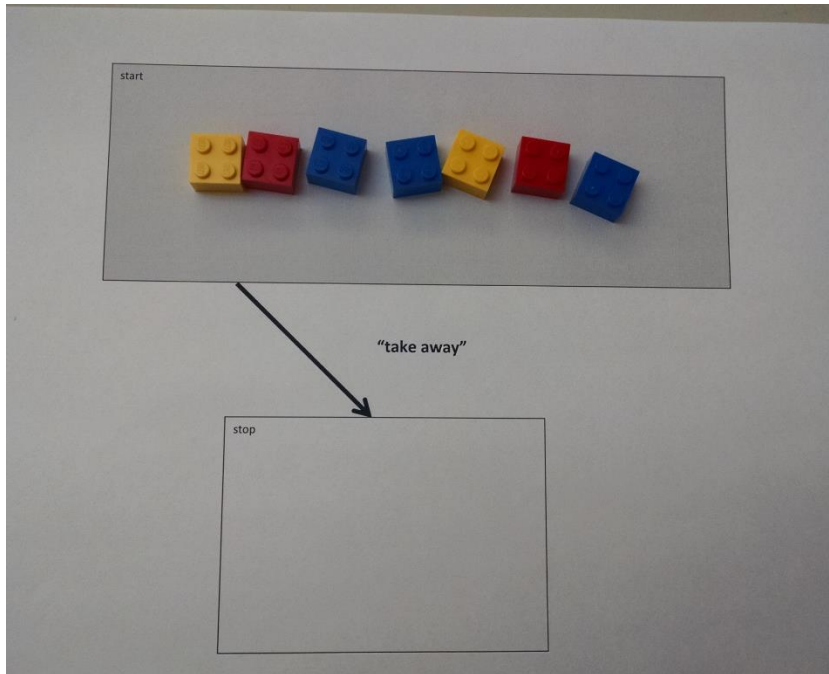
STEP 2

1. Teach student to demonstrate “pull into one pile” using the generalized mat – see 1<sup>st</sup> photo below.
2. The end result should look something like what is shown in the 2<sup>nd</sup> photo below.

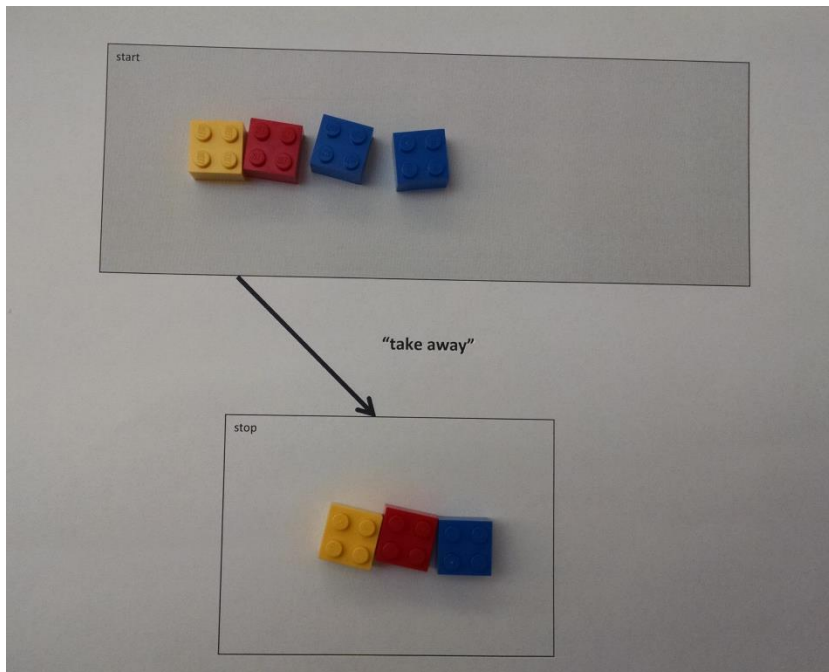


STEP 3

1. Use the "take away" mat
2. Put items in both boxes.
3. Give oral prompt to "take away"



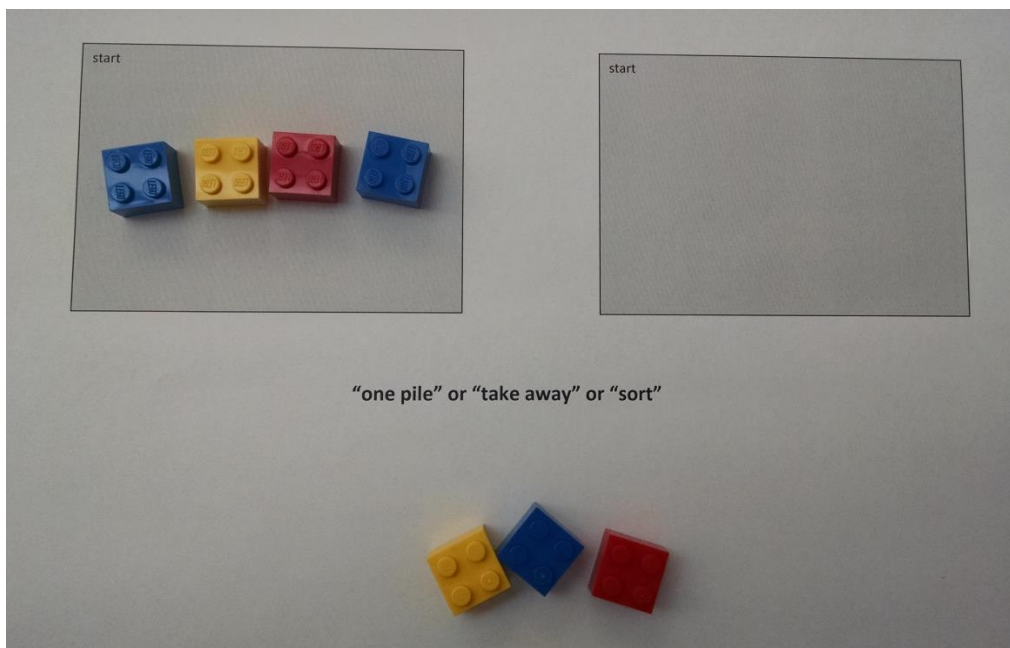
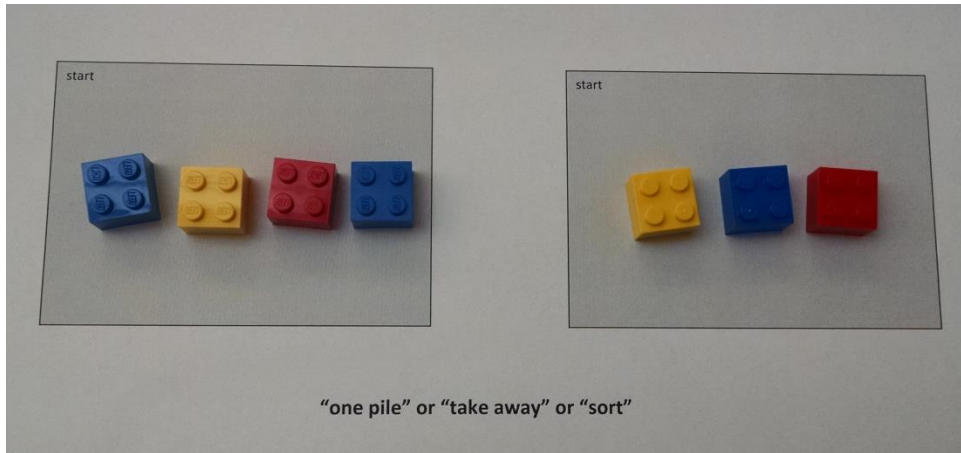
4. The end result should look like what is shown in the photo below.
5. Ask the student to find the "what's left?"





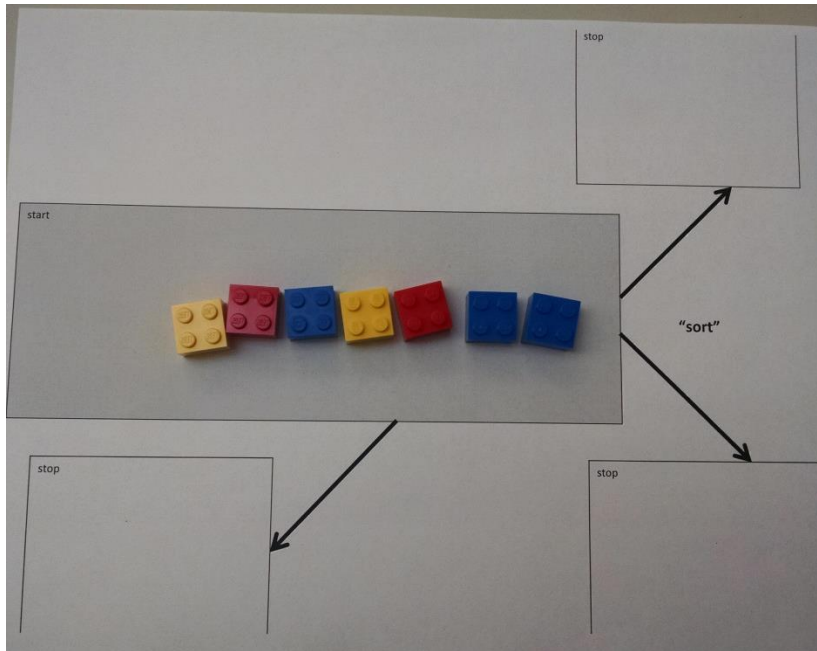
STEP 4

1. Teach student to demonstrate “take away” using the generalized mat – see 1<sup>st</sup> photo below.
2. The end result should look something like what is shown in the 2<sup>nd</sup> photo below.

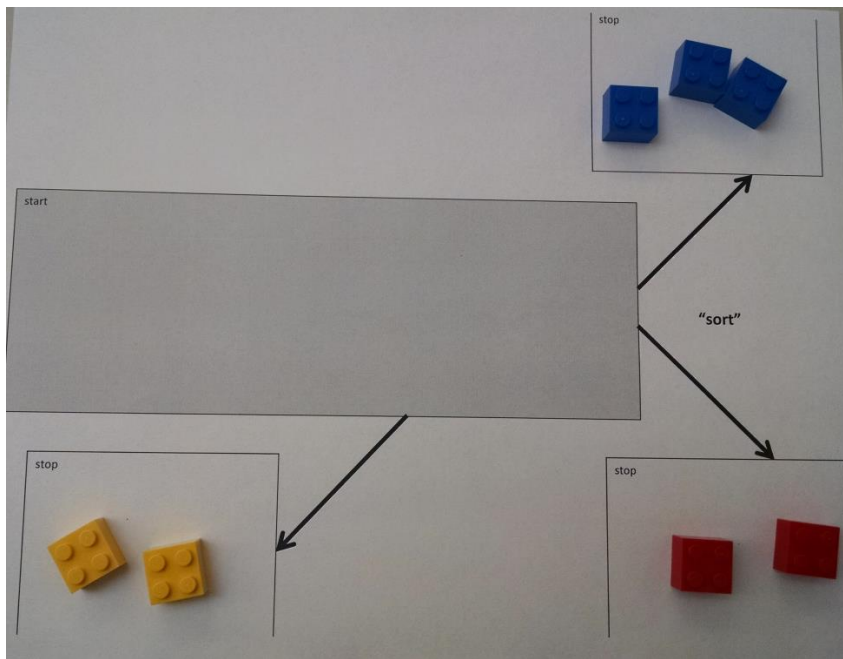


STEP 5

1. Use the "sort" mat
2. Put items in both boxes.
3. Give oral prompt to "sort"

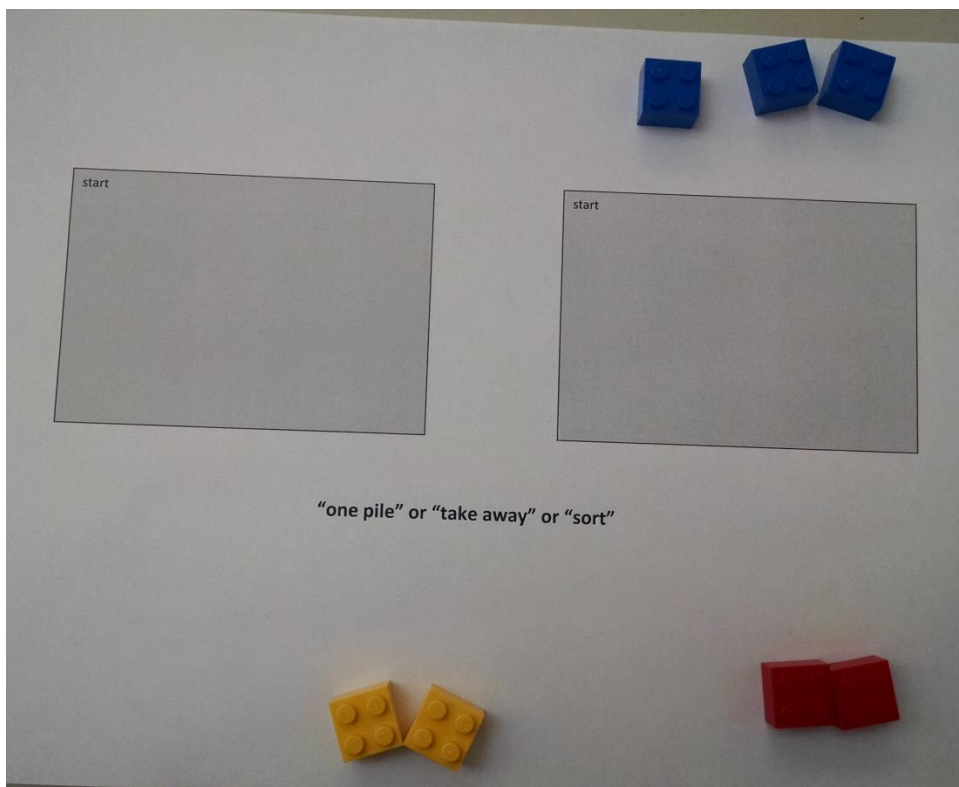
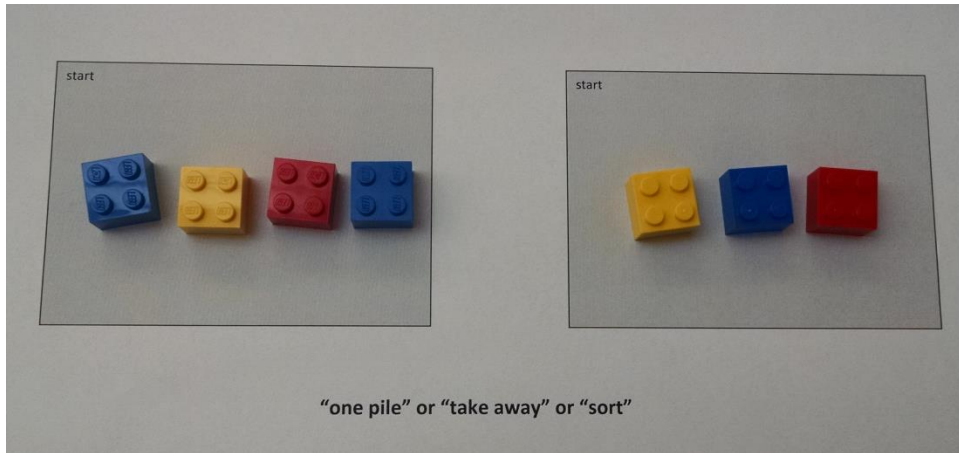


4. The end result should look like what is shown in the photo below.
5. Ask the student to explain what the groups are, e.g. "blues."



STEP 6

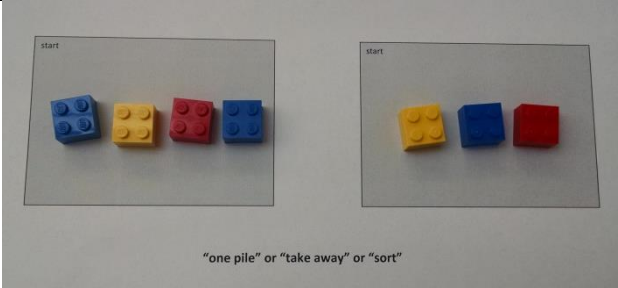
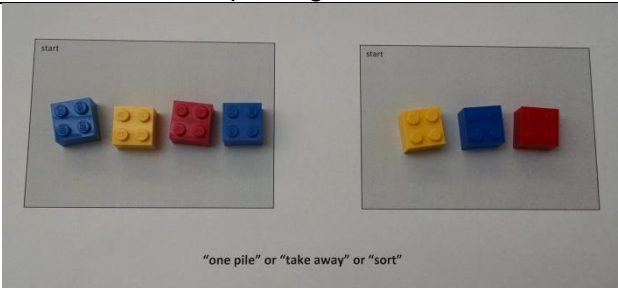
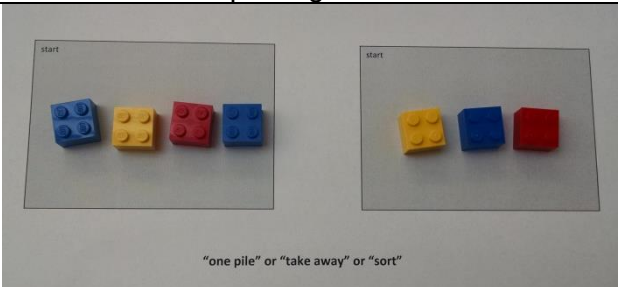
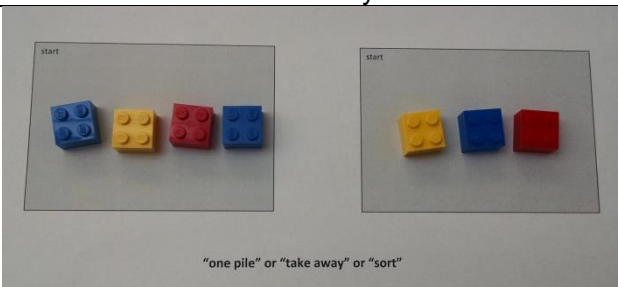
1. Teach student to demonstrate "sort" using the generalized mat – see 1<sup>st</sup> photo below.
2. The end result should look something like what is shown in the 2<sup>nd</sup> photo below.



**PHASE 2**

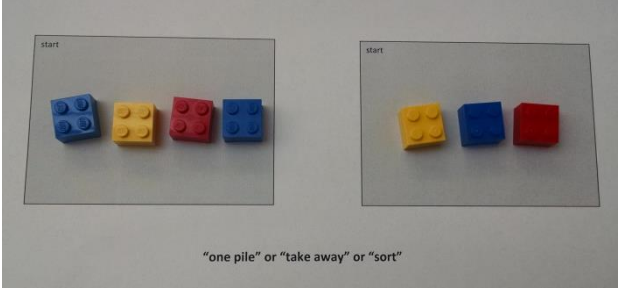
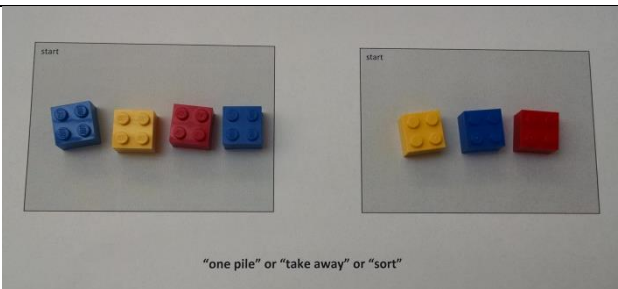
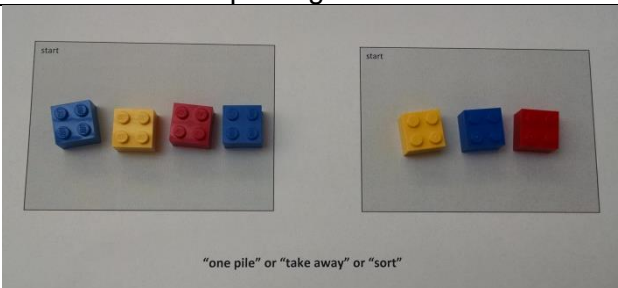
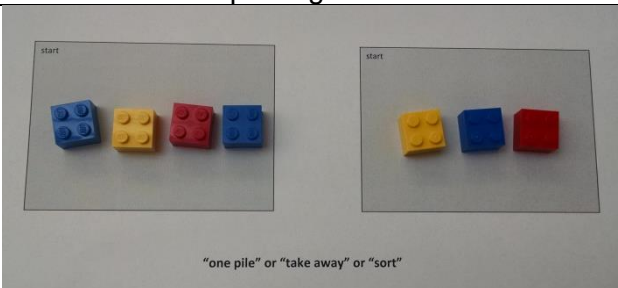
**STEP 7**

1. “pull together” and “take away” using the general mat are presented to the student randomly as trials. Below is an example.
2. The idea is that the setup is the same. The “take away” prompt is a distractor used to ensure the student doesn’t automatically pull all the items together whenever he or she sees the mat.
3. Data is collected for the trials and 100% out of 10 trials is the threshold before moving on (you certainly have flexibility in choosing the level for mastery).

<p>Trial 1</p>	 <p>“pull together”</p>
<p>Trial 2</p>	 <p>“pull together”</p>
<p>Trial 3</p>	 <p>“take away”</p>
<p>Trial 4</p>	 <p>“pull together”</p>

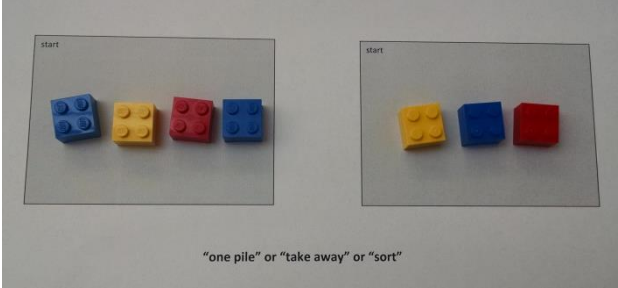
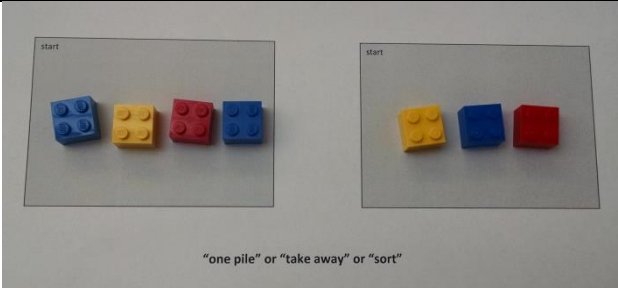
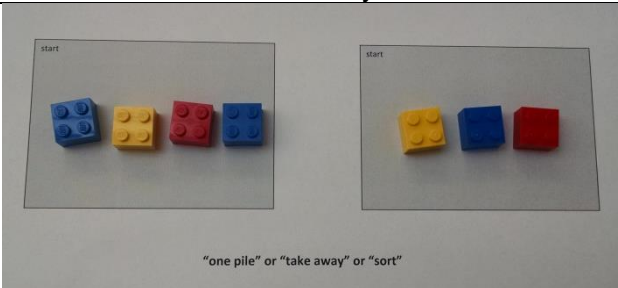
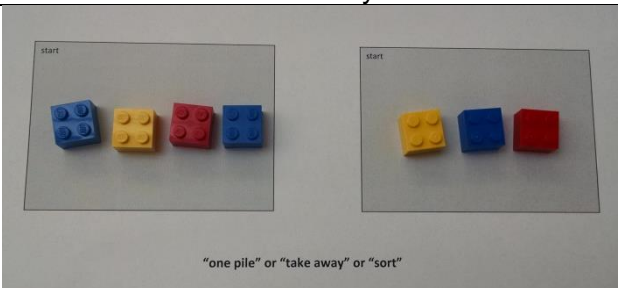
STEP 8

1. "pull together" and "sort" using the general mat are presented to the student randomly as trials. Below is an example.
2. The idea is that the setup is the same. The "sort" prompt is the distractor for this step will eventually be included in a field of 3 situation.

<p>Trial 1</p>	 <p>"one pile" or "take away" or "sort"</p> <p>"sort"</p>
<p>Trial 2</p>	 <p>"one pile" or "take away" or "sort"</p> <p>"pull together"</p>
<p>Trial 3</p>	 <p>"one pile" or "take away" or "sort"</p> <p>"pull together"</p>
<p>Trial 4</p>	 <p>"one pile" or "take away" or "sort"</p> <p>"sort"</p>

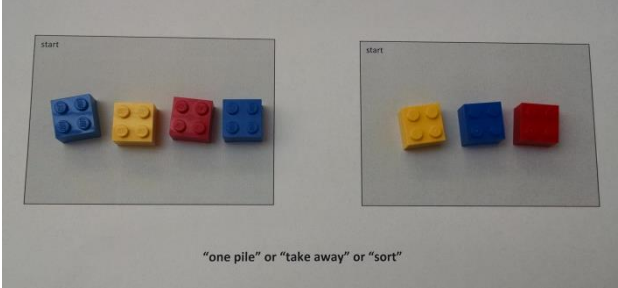
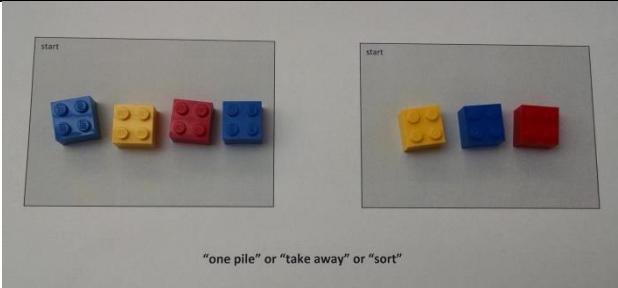
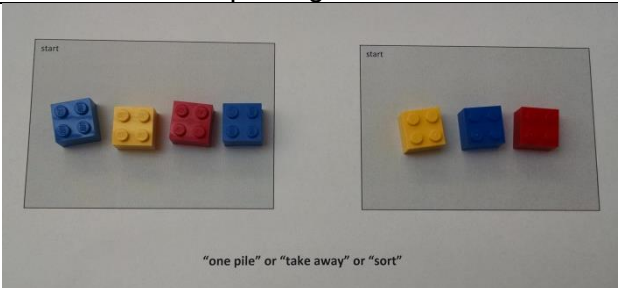
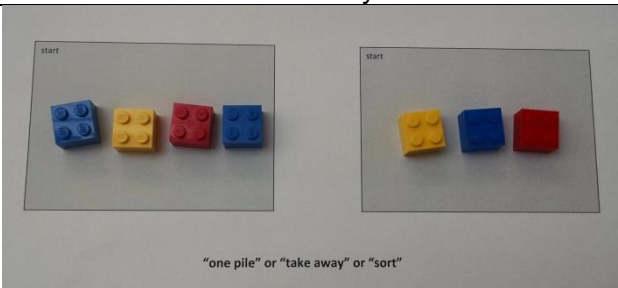
STEP 9

1. "takeaway" and "sort" using the general mat are presented to the student randomly as trials. Below is an example.
2. Even though "pull together" is not included, practice with these two as a pair prepares the student for the field of 3 when all 3 prompts are randomly used.

<p>Trial 1</p>	 <p style="text-align: center;">"sort"</p>
<p>Trial 2</p>	 <p style="text-align: center;">"take away"</p>
<p>Trial 3</p>	 <p style="text-align: center;">"take away"</p>
<p>Trial 4</p>	 <p style="text-align: center;">"sort"</p>

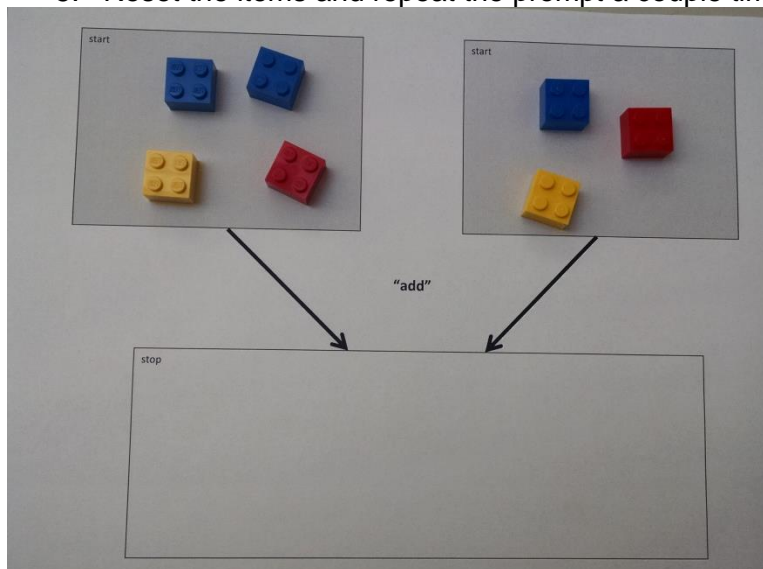
STEP 10

1. "pull together", "take away" and "sort" using the general mat are presented to the student randomly as trials. Below is an example.
2. This is the field of 3 situation used to double check that the student understands the concept of pulling together which is the concept of addition.

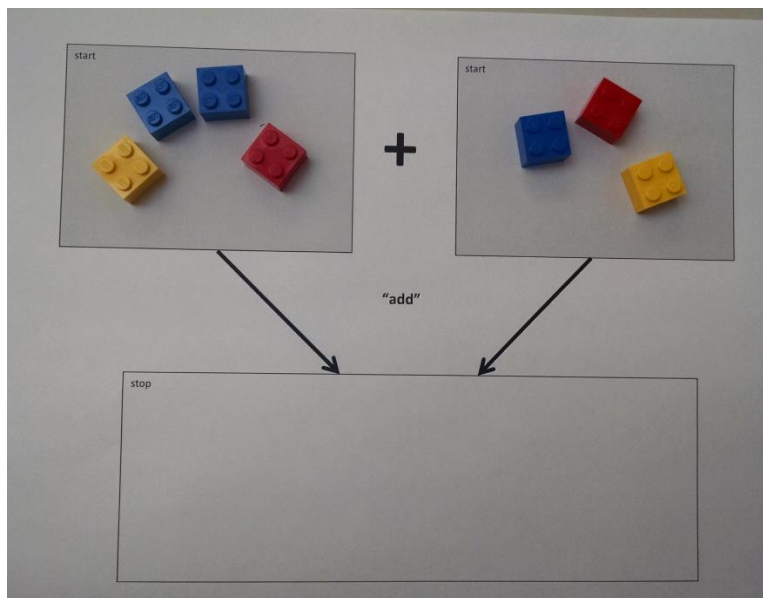
<p>Trial 1</p>	 <p>"one pile" or "take away" or "sort"</p> <p>"sort"</p>
<p>Trial 2</p>	 <p>"one pile" or "take away" or "sort"</p> <p>"pull together"</p>
<p>Trial 3</p>	 <p>"one pile" or "take away" or "sort"</p> <p>"take away"</p>
<p>Trial 4</p>	 <p>"one pile" or "take away" or "sort"</p> <p>"sort"</p>

**PHASE 3****STEP 11**

1. Use the “add” mat
2. Put items in both boxes.
3. Give oral prompt to “pull into one pile”
4. Explain this means to “add”
5. Reset the items and repeat the prompt a couple times.



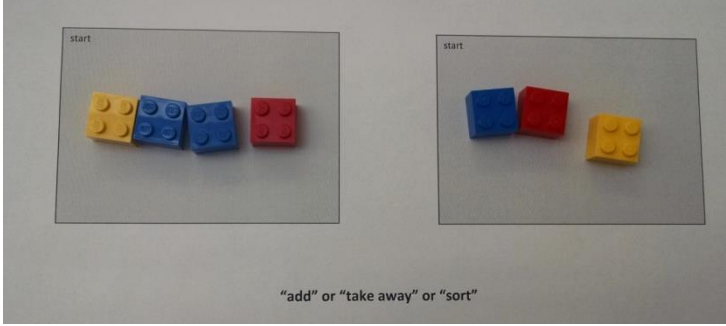
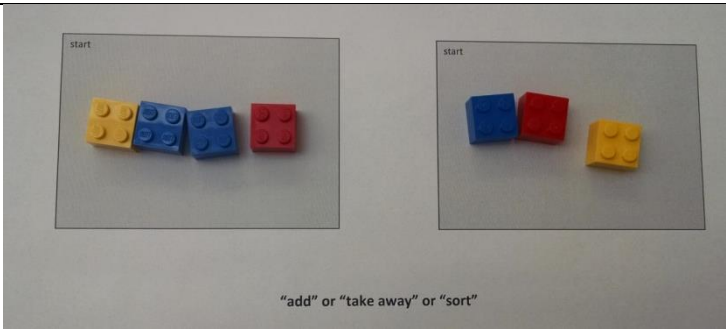
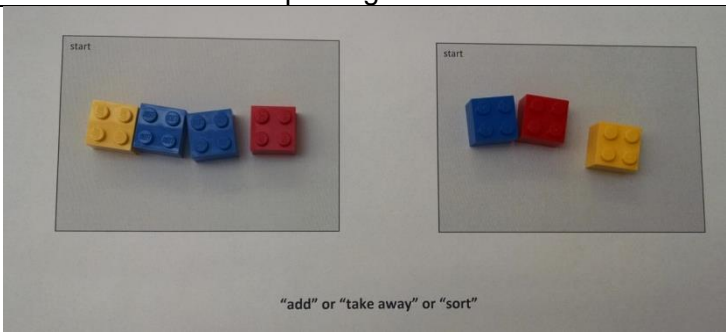
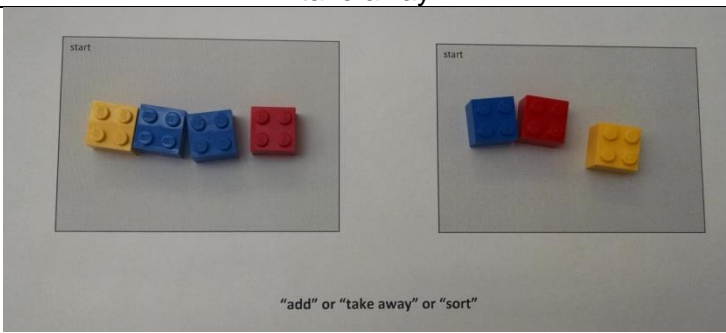
6. Repeat steps with the “+” mat

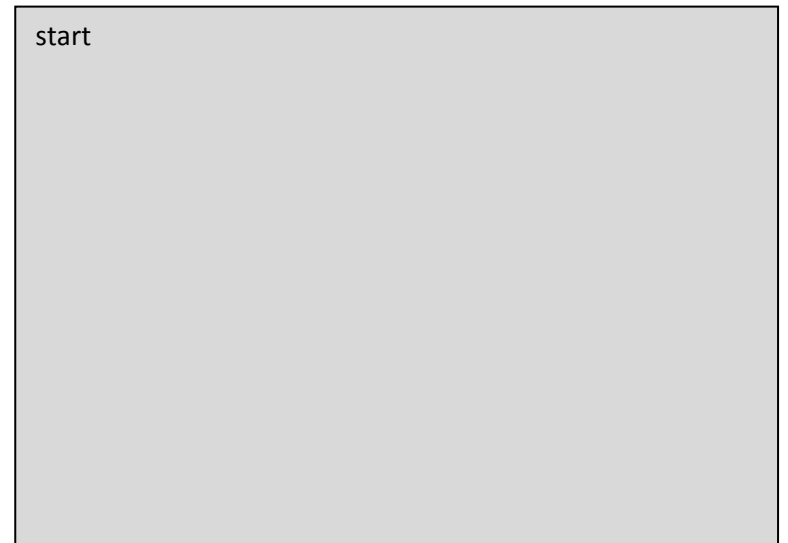




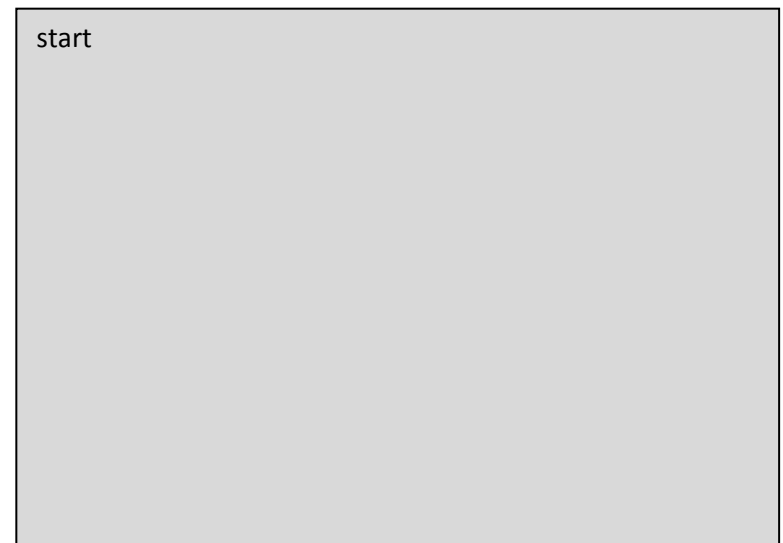
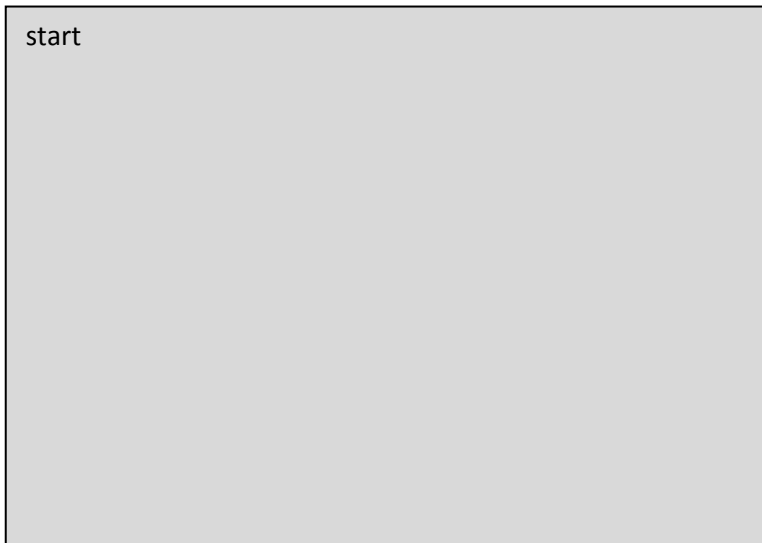
STEP 12

1. Redo the field of 3 prompts using the generalized “add” mat.
2. After completing this to mastery replace the verbal “add” prompt by showing a card with “+” on it.  
You can say “add” and show the card then eventually just show the card.

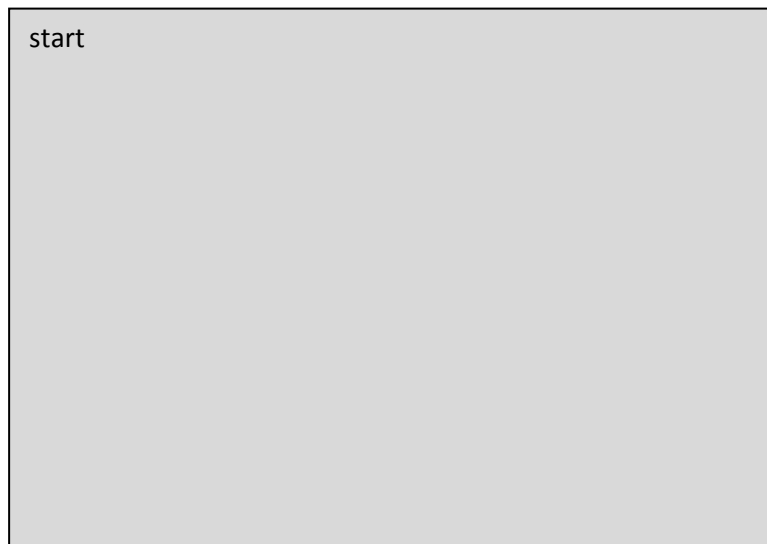
<p>Trial 1</p>	 <p>“sort”</p>
<p>Trial 2</p>	 <p>“pull together”</p>
<p>Trial 3</p>	 <p>“take away”</p>
<p>Trial 4</p>	 <p>“sort”</p>



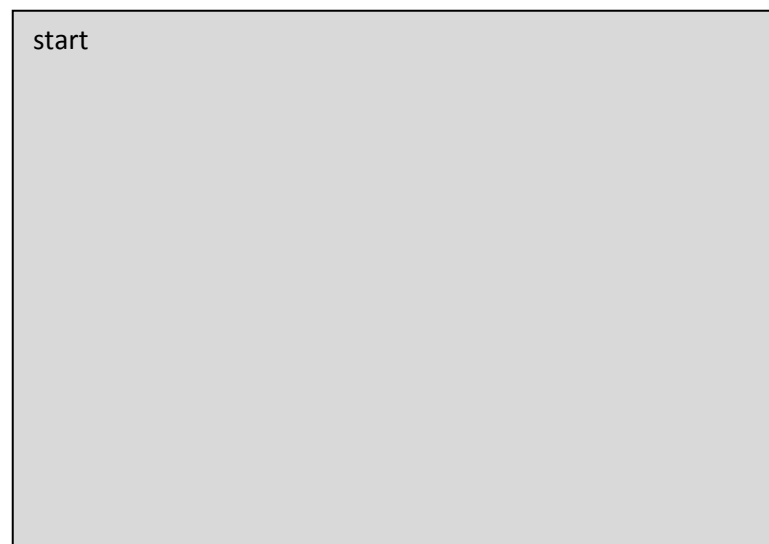
**“add” or “take away” or “sort”**



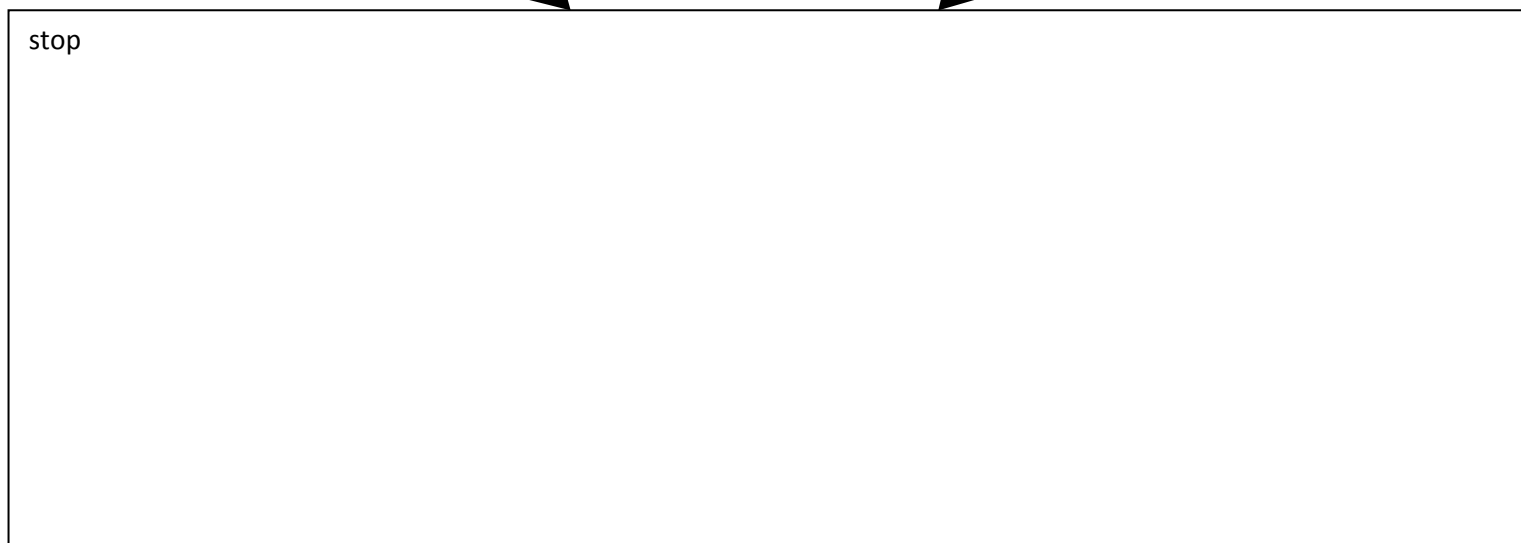
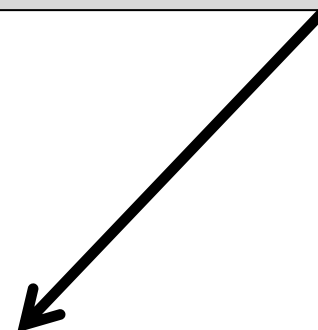
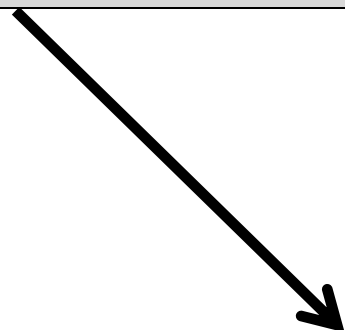
**“one pile” or “take away” or “sort”**

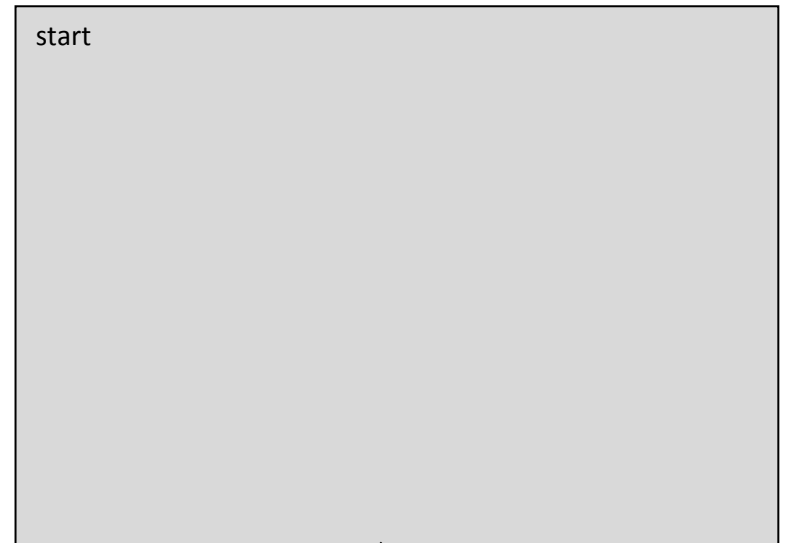


**+**

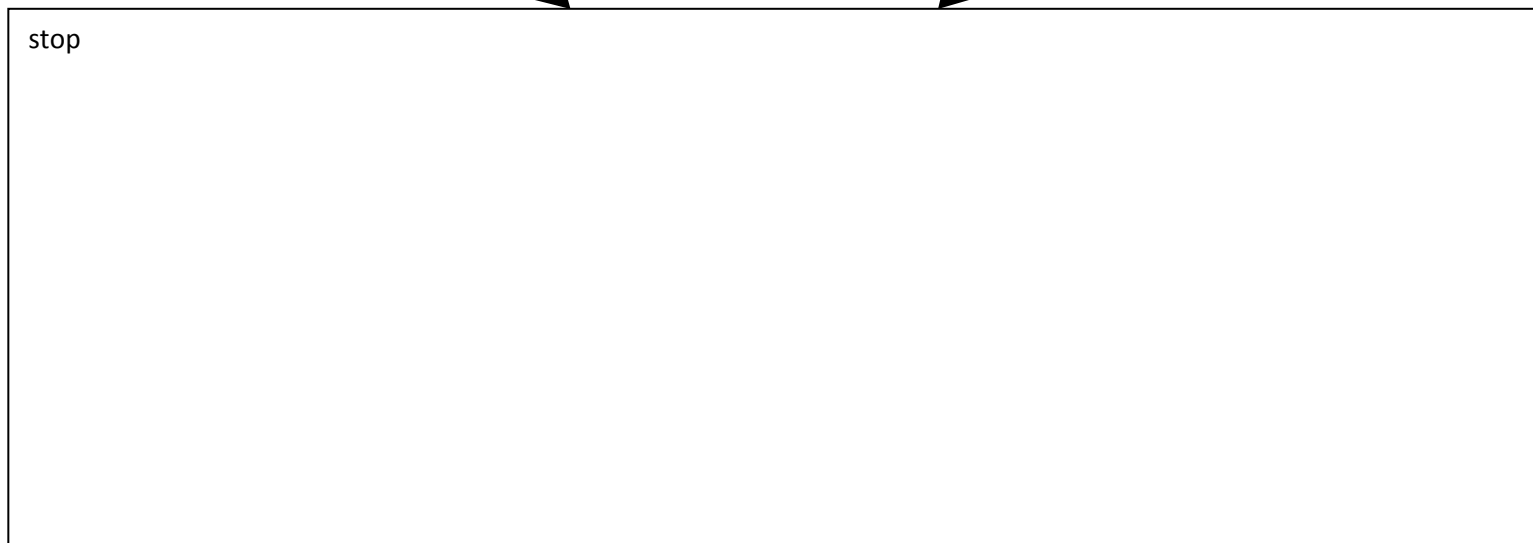


**"add"**



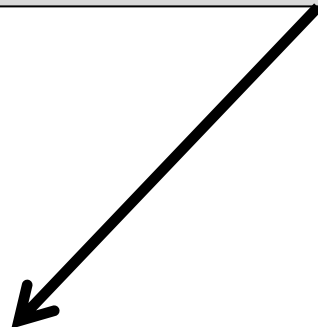
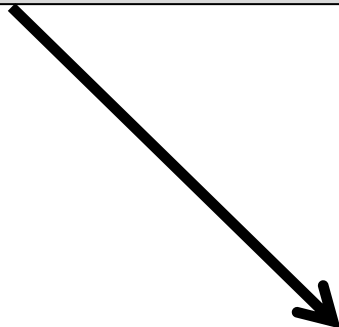
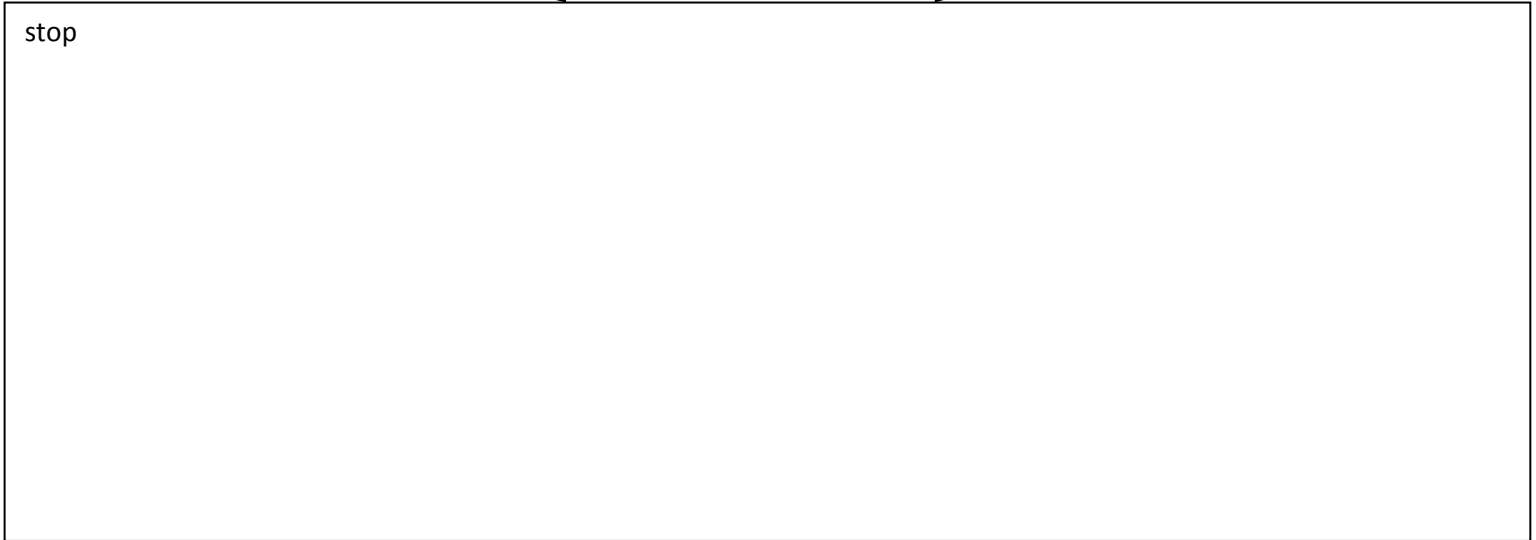


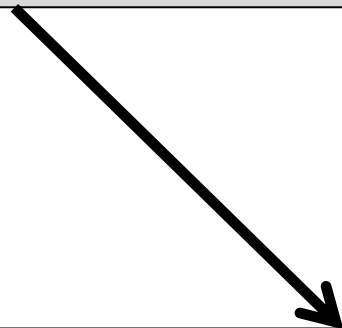
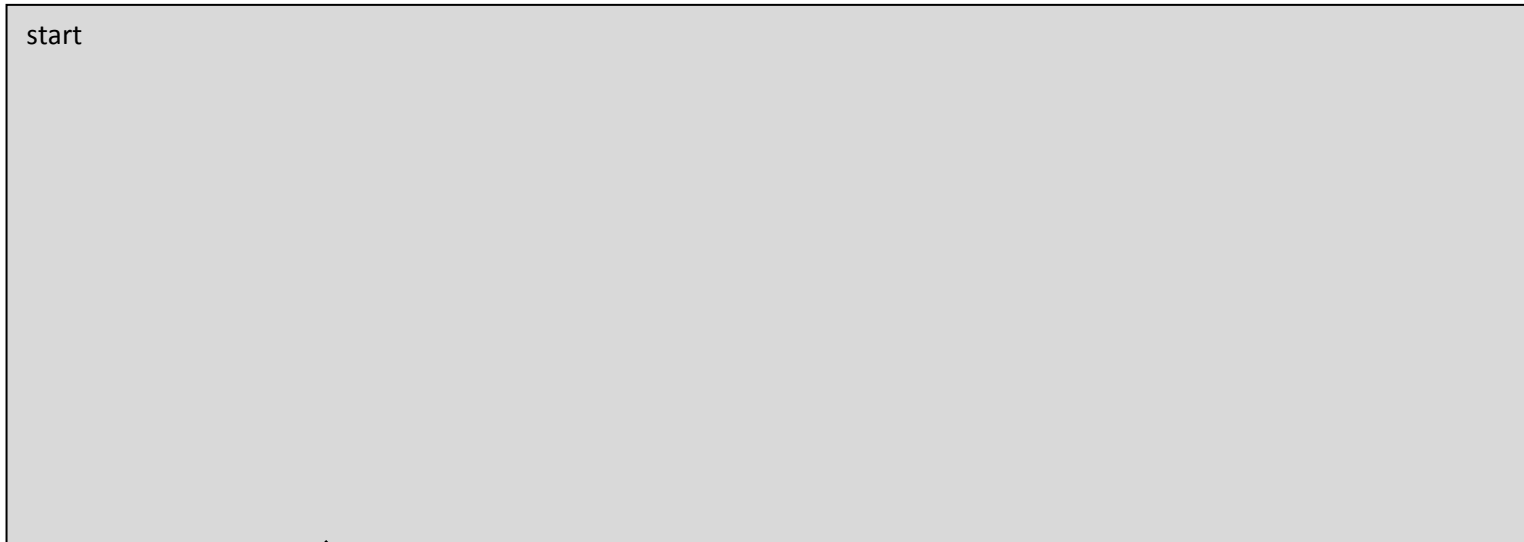
**“one pile”**





“add”





**“take away”**



Teaching the Concept of Addition

