CORE MATH®

Five Favorite Strategies for Teaching About Fractions

Dean Ballard

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Icons in the Session Today

Chat

- Enter response into Chat.

On Your Own

- On your own paper.

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Objectives

- See **physical and visual models** that are flexible, doable, and clearly connect fraction concepts.
- Recognize **connections** between **fraction** concepts and **whole number** concepts.
- Learn how **fraction concepts build on each other** in sensible ways.
- Recognize the **number line** as a key tool for understanding fraction concepts.
- Experience **challenge problems** with fractions that extend and assess student understanding.
- Gain ideas for **fluency building activities** that are fun and effective.

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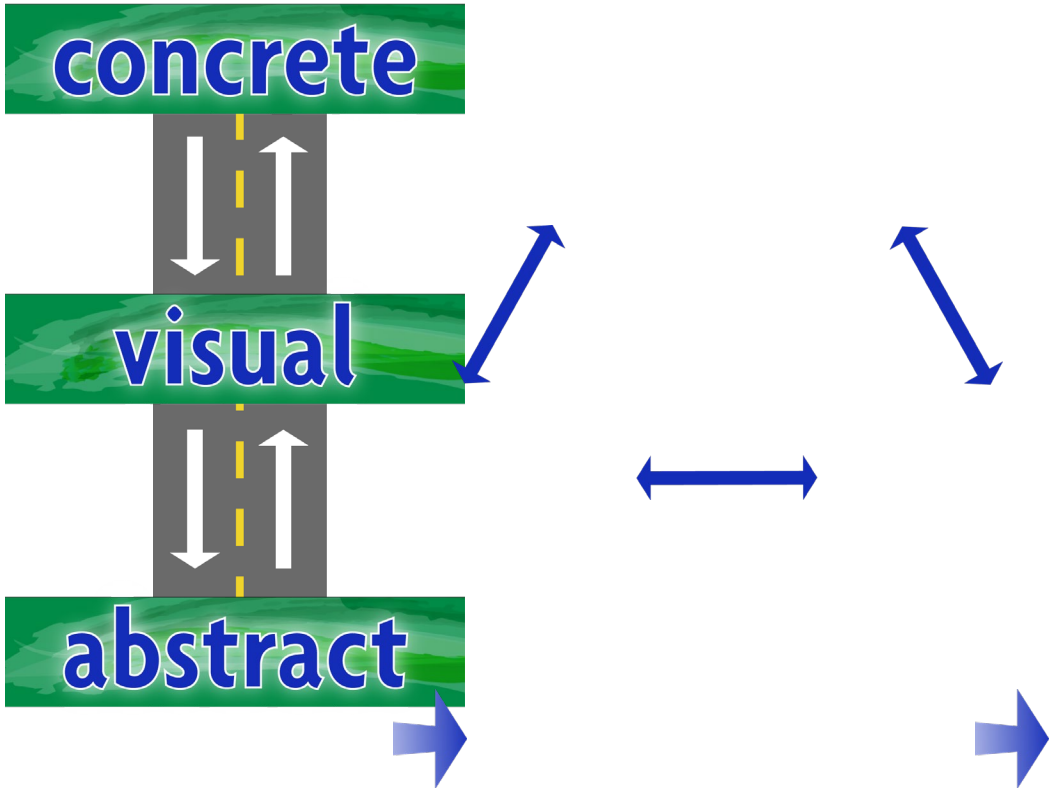
Progression of Learning
Concrete – Visual - Abstract



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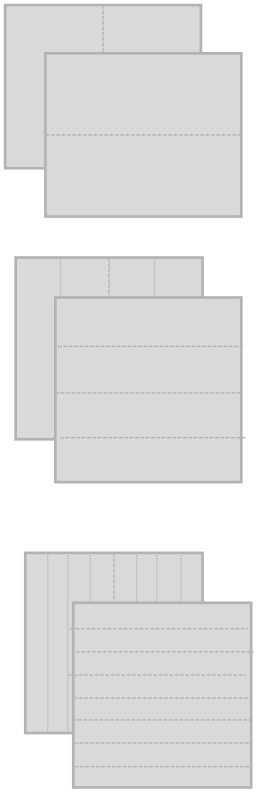
Paper Folding 1

- Students work in pairs. Each student has three pieces of letter size paper (8½” by 11”).
- Fold one paper in half. What does this make? Record.
  - One student fold vertically, and one fold horizontally
- Fold another paper in half twice. What does this make?
  - One student fold vertically, and one fold horizontally
- Fold another paper in half 3 times. What does this make?
  - One student fold vertically, and one fold horizontally
- Compare / discuss, how many fourths make a half? How many eighths make a half? How many eighths make a fourth? Are you and your partner’s halves equal? Why?



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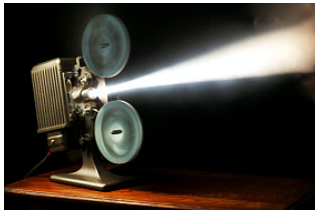
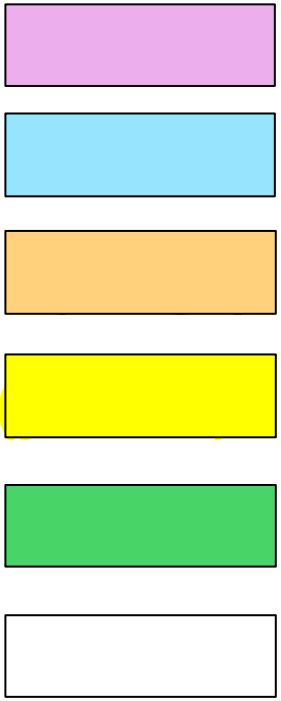
Visualize Fractions with Paper Folding



1. Teach using visual representations. Write the fraction on these strips in words on one side, in symbols on the other side.
   - Fold the pink paper strip into halves
   - Fold the blue paper strip into fourths
   - Fold the beige paper strip into eighths
   - Fold the yellow paper strip into thirds
   - Fold the green paper strip into sixths
   - Fold the white paper strip into fifths



VIDEOS

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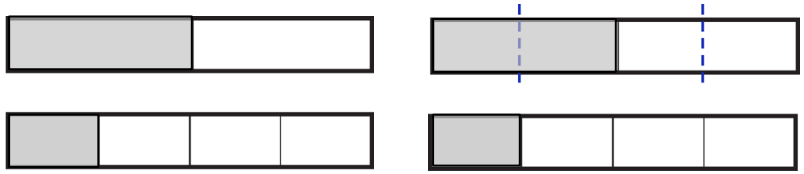
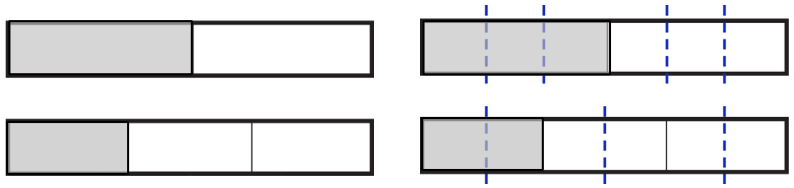
Comparing Fractions with - Tape Diagrams

How can I change each tape diagram so that both have the same-size parts?

$\frac{1}{2}$ and $\frac{1}{4}$



$\frac{1}{2}$ and $\frac{1}{3}$



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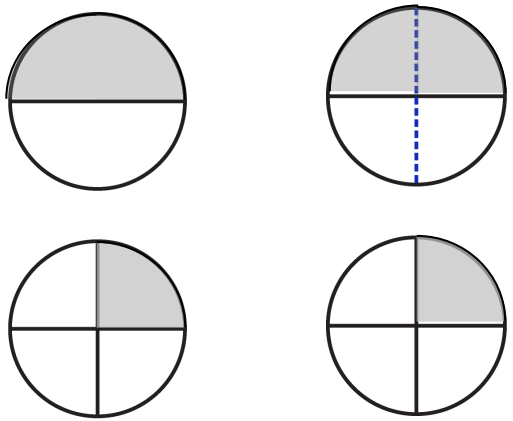
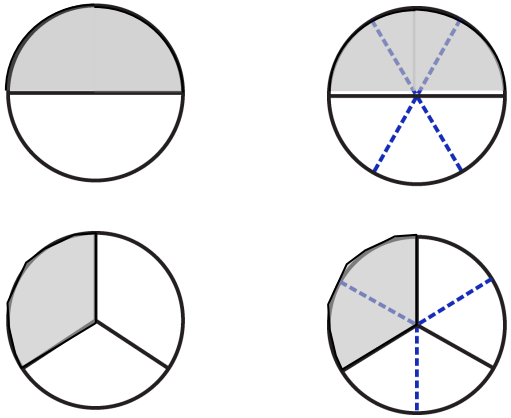
Circle Diagrams

How can I change each circle diagram so that both have the same-size parts or the same **UNITS**?

$\frac{1}{2}$ and $\frac{1}{4}$



$\frac{1}{2}$ and $\frac{1}{3}$



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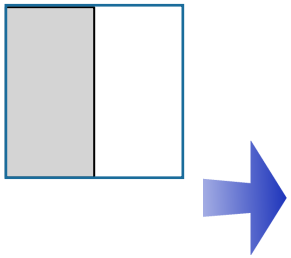
Using Area Models to Visualize Multiplication

¼ x ½

¼ of ½



$$\frac{1}{4} \times \frac{1}{2} = \frac{1}{4 \times 2} = \frac{1}{8}$$

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{2 \times 4} = \frac{1}{8}$$

When we are multiplying ½ by ¼ we are **dividing** each half of the whole by four. **So, why do we multiply the denominators?**

We are creating **four times** as many parts!

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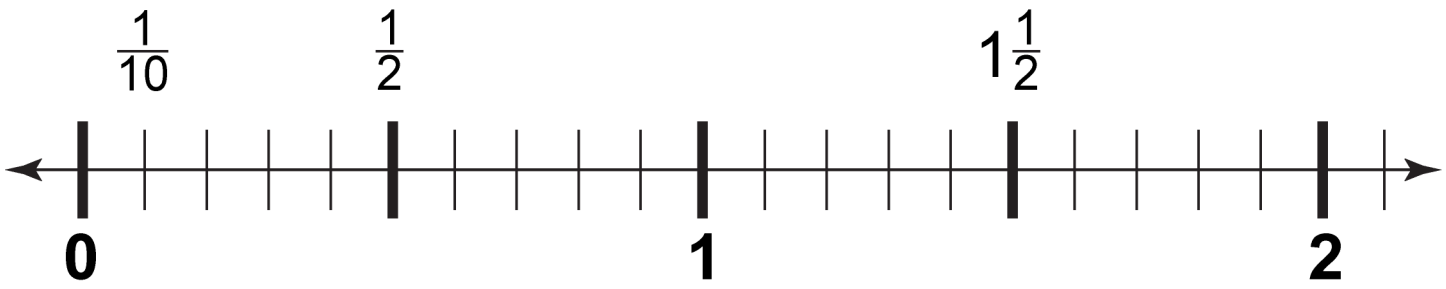
Number Lines - Fractions as Numbers

A fraction is a representation of a number. As such it can be placed on the number line.



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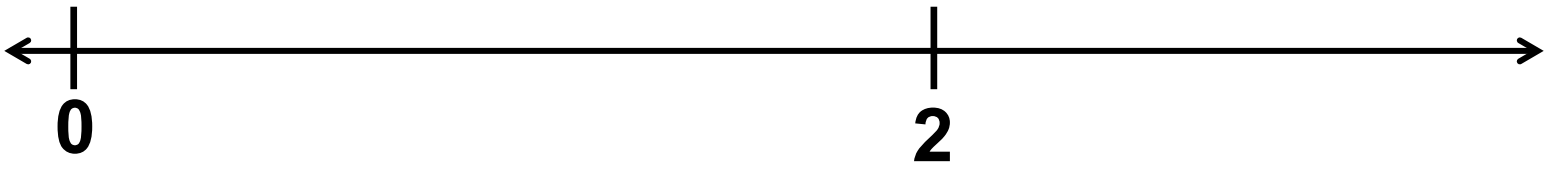
Number Lines - Fractions as Numbers

On Your Own

$$\frac{8}{5} \quad \frac{3}{3} \quad \frac{1}{4} \quad \frac{1}{2}$$

Draw this number line as shown below on your paper.
Place the four fractions shown above on the number line.



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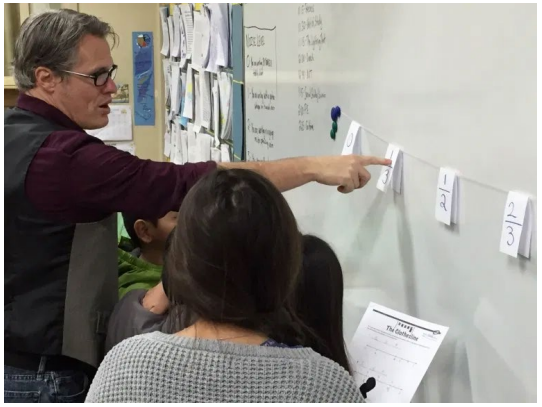
Clothes Line Number Lines with Fractions

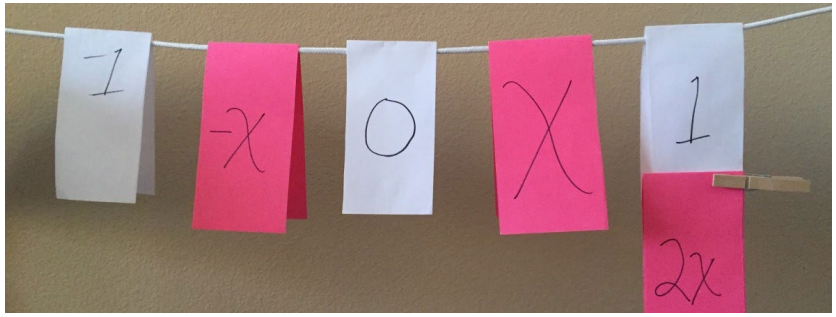
FractionTalks.com

Chase Orton
UndercoverCalculus.com



PREVIEW

ClothesLineMath.com



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Online Visual Tools

The MATH LEARNING CENTER

https://www.mathlearningcenter.org/apps

FRACTIONS

Represent fractions with denominators from 1 to 100 with circle and bar models.

NUMBER LINE

Visualize and work with numbers in sequence on a virtual number line with or without tick marks.

PREVIEW

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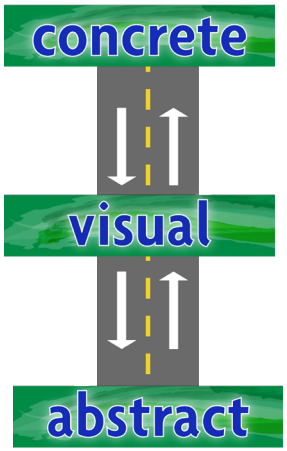
Wrap Up – Concrete – Visual - Abstract

Importance of using concrete and visual models connected with and leading to abstract or symbolic numerical representations of fractions, fraction properties and operations with fractions.

- Paper folding activities
- Rectangular diagrams/tape diagrams
- Number lines (& clothesline number lines)
- Online visual tools (such as *The Math Learning Center apps*)



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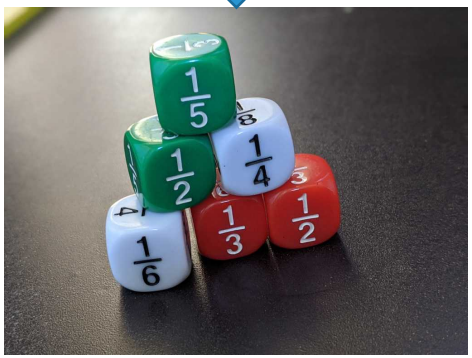
Connecting Fractions with Whole Numbers

- Concept of Units
- Fractions as Numbers
- Equivalent Fractions
- Addition and Subtraction
- Multiplication and Division



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Directions in Standards for Connections

Grade 3	Grade 4	Grade 5	Grade 6
• Unit fractions • Part-whole • Equal parts • Same size wholes • Fractions as numbers • Compare fractions • Use visual models	• Equivalent fractions • Use unit fractions to compose and decompose fractions • Use previous understandings with operations to understand addition, subtraction, and multiplication of fractions • Use visual models • Solve word problems for addition and subtraction • Decimal fractions	• Apply understanding of fractions to add and subtract with unlike denominators • Fluency with addition and subtraction • Estimate sums and differences • Use previous understandings with operations to understand multiplication and division • Make sense of multiplication and division • Solve word problems for addition, subtraction multiplication, and division • Use visual models	• Compute quotients with fractions. • Interpret quotients • Solve word problems with fraction operations • Use visual models • Use equations

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UNITS – From Place Value to Fractions

Place Value

thousands, hundreds, tens,

ONES,

tenths, hundredths, thousandths

Units

- Units – ones, tens, hundreds, thousands . . .
- Units – tenths, hundredths, thousandths . . .
- Units – halves, thirds, fourths, fifths, tenths …

Denominator - describes the **unit** with fractions, based on partitioning the "ones" unit.

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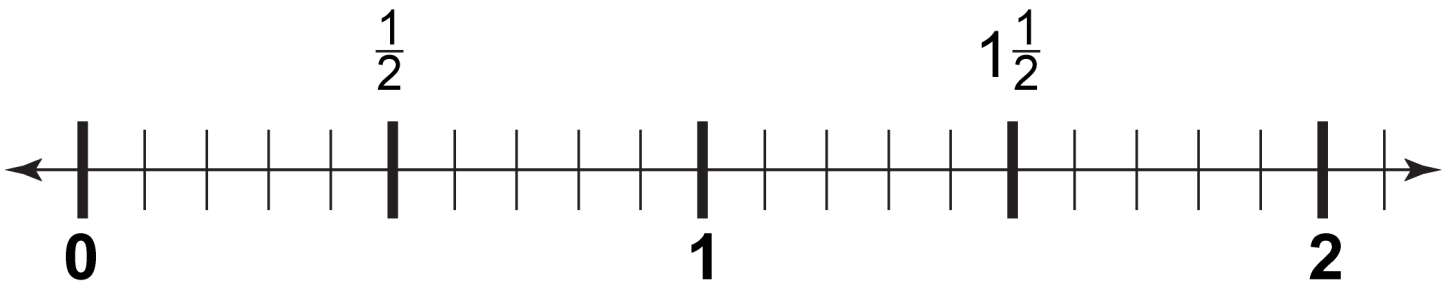
Number Lines - Fractions as Numbers

A fraction is a representation of a number. As such it can be placed on the number line.



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Equivalent Fractions

Multiplicative identity property:

Any number x 1 = the same number

18 x 1 = 18

Same property is central to creating equivalent fractions.

$$\frac{2}{5} \times 1 = \frac{2}{5}$$

However, the end result doesn't always look the same.

Suppose we multiply 2/5 by 3/3? The results looks like a different value.

$$\frac{2}{5} \times \boxed{1} = \frac{3}{3} = \frac{6}{15}$$

Use prior visual models, and talk about multiplying by the "big bad one."

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Multiple Equivalent Representations

Chat

Show another way to numerically represent 451 besides as (4 hundreds + 5 tens + 1 one).

- 4 hundreds + 4 tens + 11 one
- 3 hundreds + 15 tens + 1 one
- 3 hundreds + 14 tens + 11 ones

$$\begin{array}{r} 451 \\ -\ 273 \\ \hline \end{array}$$

Show another way to numerically represent $\frac{2}{5}$.

- 4/10
- 8/20
- 6/15

$$\frac{2}{5} - \frac{2}{15} = \frac{6}{15} - \frac{2}{15}$$

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Addition and Subtraction – Like Units

Whole numbers:

Combine like units

- Ones with ones
- Tens with tens
- Hundreds with hundreds

$$\begin{array}{r} 451 \\ +\ 243 \\ \hline 694 \end{array}$$

Fractions:

Combine like units (denominators are the units!)

- Fourths with fourths,
- Fifths with fifths,
- Fifteenths with fifteenths . . .

$$\frac{2}{5} - \frac{2}{15} = \frac{6}{15} - \frac{2}{15} = \frac{4}{15}$$

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Emphasize and Name the Units

Whole numbers:

40 + 50 = 90 → 4 tens + 5 tens = 9 tens

Fractions:

1/5 + 3/5 = 4/5 → 1 fifth + 3 fifths = 4 fifths

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Multiplication

Multiplication of Whole Numbers:

$$3 \times 5 = 5 + 5 + 5 = 15$$

Multiplication of Fractions:

$$3 \times \frac{1}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5}$$

Multiplication of Fractions:

$$3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5}$$

$$\frac{3 \times 2}{5} = \frac{6}{5}$$

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Division – How Many In the Group?

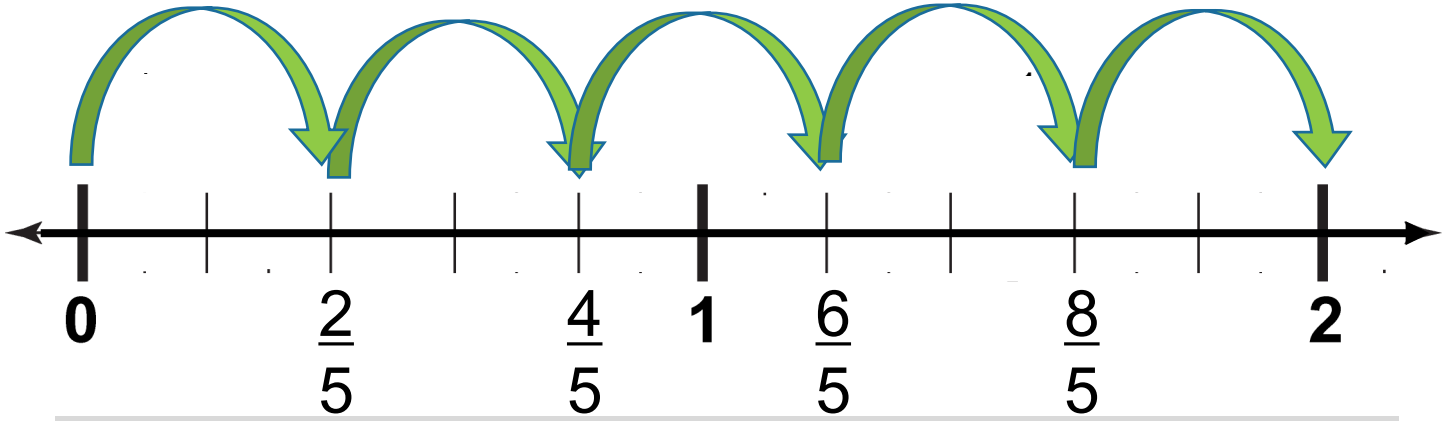
Division with Whole Numbers:

12 divided by 4 → How many fours are in 12?

- There are three 4s in 12. 12 divided by 4 = 3.

Division with Fractions:

Two divided by $\frac{2}{5}$ → How many $\frac{2}{5}$ are in 2?



There are five $\frac{2}{5}$ in 2. Therefore 2 ÷ $\frac{2}{5}$ = 5.

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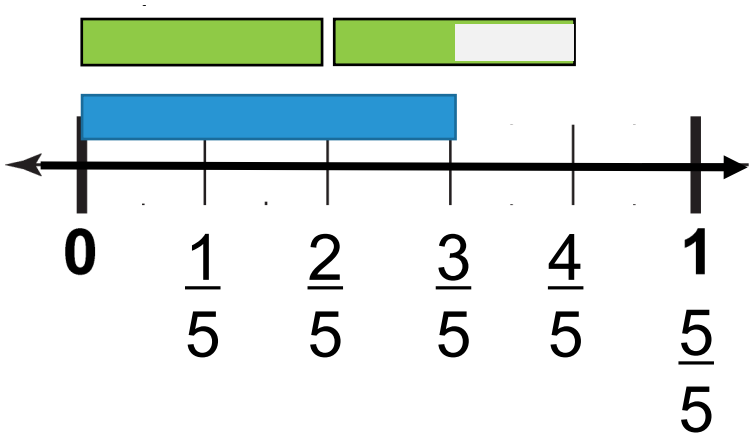
Batches of Muffins

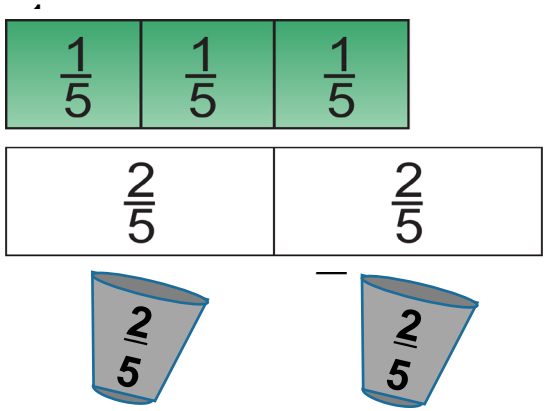
I have $\frac{3}{5}$ cup of sugar left in my sugar container. I need $\frac{2}{5}$ of a cup of sugar for each whole batch of muffins.

How many batches of muffins can I make?

$$\frac{3}{5} \div \frac{2}{5} = \text{number of batches of muffins}$$

There are $1\frac{1}{2}$ two-fifths in three-fifths. Therefore $\frac{3}{5} \div \frac{2}{5} = 1\frac{1}{2}$.





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Wrap Up – Connecting Fractions with Whole Numbers

- **Concept of Units** – the denominator is the unit based on some part of the ”ones” unit.
- **Fractions as Numbers** – fit on a number line just like whole numbers and can be used to count parts of objects.
- **Equivalent Fractions** – Big Idea that numbers can be represented in many equivalent forms, and we use different versions based on need.
- **Addition and Subtraction** – Combine like units
- **Multiplication and Division** - Repeated addition, area model, how many of one quantity is in the other quantity.

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Connections Among Fraction Concepts

- Building fractions from unit fractions
- Patterns in division of fractions

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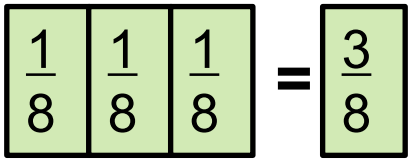
Building Fractions from Unit Fractions

Use unit fractions to compose and decompose fractions. (CCSSM 2010)

- With whole numbers we build on the "ones" unit.
- With fractions we build other fractions from unit fractions (fractions with a numerator of one).



$$\frac{3}{8} = 3\left(\frac{1}{8}\right) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{8} + \frac{2}{8}$$

$$\frac{11}{8} = \frac{8}{8} + \frac{3}{8} = 1 + \frac{3}{8} = 1\frac{3}{8}$$

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Patterns with Fraction Division

- Fill in the tables with the correct quotients.
- Describe any patterns you notice.
- In **CHAT** describe a shortcut for the pattern you see in the table.

dividend

divisor

Expression	Quotient
$3 \div 3$	
$3 \div 1$	
$3 \div \frac{1}{2}$	
$3 \div \frac{1}{3}$	
$3 \div \frac{1}{4}$	
$3 \div \frac{1}{5}$	
$4 \div \frac{1}{4}$	

Describe the pattern in Chat

Expression	Quotient
$3 \div \frac{1}{4}$	
$3 \div \frac{2}{4}$	
$3 \div \frac{3}{4}$	
$6 \div \frac{1}{4}$	
$6 \div \frac{2}{4}$	
$6 \div \frac{3}{4}$	

Describe the pattern in Chat

Dividing a whole number by a fraction is the same as

- multiplying the whole number by the denominator of the fraction; and then
- dividing this answer by the numerator of the fraction.

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Connecting Concepts with Multiplication and Division

Divide	Multiply
$12 \div 3$	
$12 \div 4$	
$12 \div 6$	

Dividing a whole number by a fraction is the same as multiplying the whole number by the reciprocal of the fraction.

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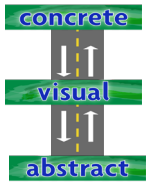
Learning About Fractions – So Far



- Concrete – Visual – Abstract
- Connecting fractions to whole numbers concepts

- Connecting fractions to other fraction concepts

  - Building on unit fractions
  - The division algorithm through patterns

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Closest to $\frac{1}{2}$

Which fraction has a value closest to $\frac{1}{2}$?

A. $\frac{5}{8}$ B. $\frac{1}{6}$ C. $\frac{2}{2}$ D. $\frac{1}{5}$

Which is the most popular incorrect answer?

4th-Grade NAEP, 2009

25% answered correctly (A)

40% chose C

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Thinking about Division

On Your Own

6-8: Examine the four division problems shown below.

Without calculating the quotients, which quotient is closest to 1? Explain and/or show your reasoning.

A. $\frac{19}{20} \div \frac{1}{18}$

B. $\frac{1}{20} \div \frac{1}{18}$

C. $\frac{1}{4} \div 4$

D. $4 \div \frac{1}{4}$

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Comparing Fractions Without Using Common Denominators (or decimals or percent)

Compare the following pairs of fractions **without** converting to common denominators, common numerators, decimals, or percents, or using a number line. Pick one and explain your reasoning in Chat.

1. $\frac{3}{7} < \frac{5}{8}$

$\frac{3}{7}$ *is less than half*

$\frac{5}{8}$ *is greater than half*

2. $\frac{5}{6} > \frac{5}{8}$

Sixths are greater than eighths (same size wholes)

3. $\frac{5}{6} > \frac{3}{4}$

$\frac{5}{6}$ *is* $\frac{1}{6}$ *from* 1

$\frac{3}{4}$ *is* $\frac{1}{4}$ *from* 1

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Develop Fluency Through Engaging Activities

- Card Games
- Counting Activities

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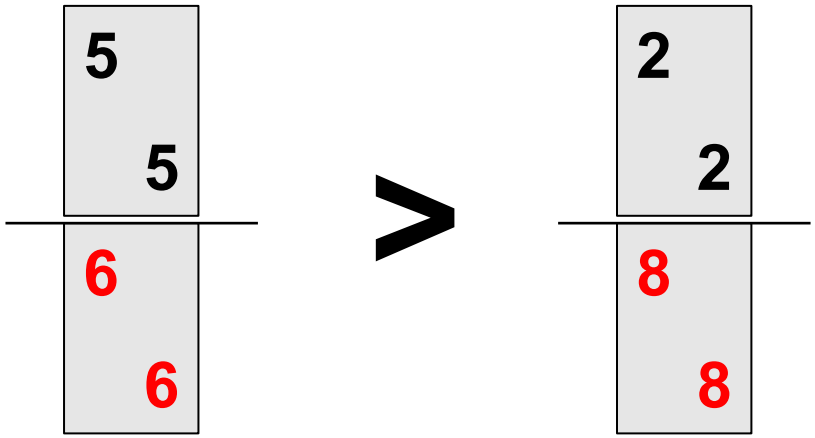
Card Game – Fraction War

- Standard deck of cards
- Divide cards up between two players
- Each player mixes his/her cards face down in a stack.
- Each player turns over her/his first two cards.
- Each player uses their own two cards to create a fraction equal to or less than one.
- The player with the greater fraction wins the round.
- Optional: Record fractions on a play sheet.



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Counting Up and Down with Fractions

- Count by halves starting at 1
- Count by halves starting at 4½.
- Count by halves starting at ¼ (1¾, 2¼, 2¾, 3¼, 3¾, 4¼, 4¾)

Advice:

- Start small with very doable numbers
- Use very clear hand signals for counting up and down
- Focus on where students are at to move forward
- Require students to stay with your hand signals
- Go back and forth across whole numbers
- **Discuss patterns and challenges**
- **Use a number line to visualize & build understanding**

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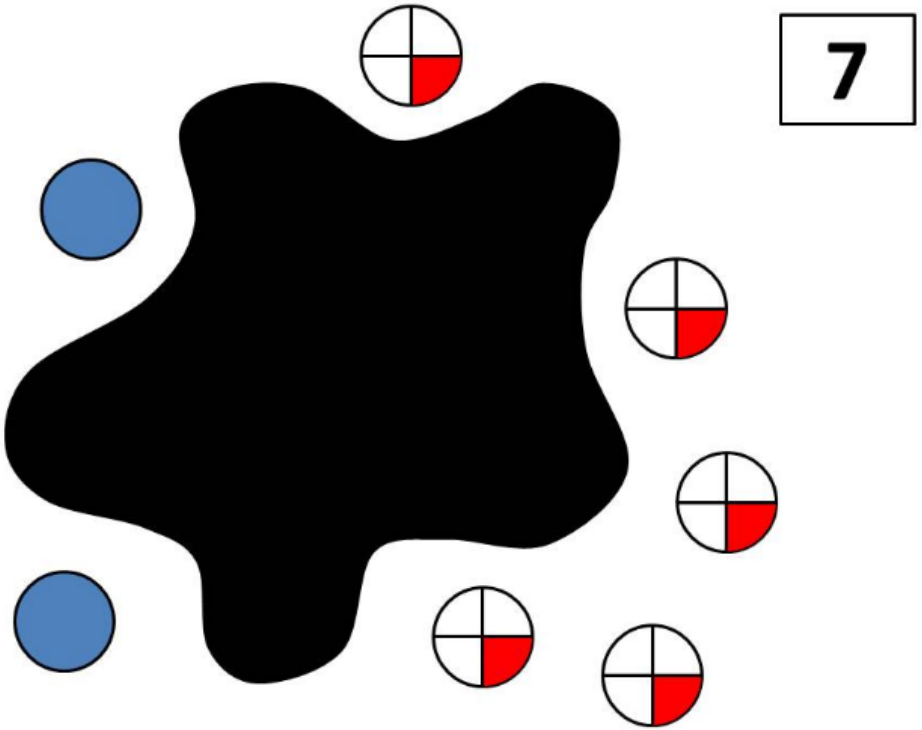
Fraction Splats! By Steve Wyborney

The Fraction Splat! Series

By Steve Wyborney | March 8, 2017 | 42



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Ideas We Explored Today

- Concrete – Visual - Abstract
- Connect fraction concepts with whole number concepts.
- Build fraction concepts on other fraction concepts
- Recognize the number line as a key tool
- Challenge problems to make us think.
- Engaging activities to build fluency

Antelope Canyon, Navajo Reservation

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Pdf of Slides

- Dropping into Chat now.
- Also, putting in Chat a link you can use to make a copy to your own google drive.

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Thank you!

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Feedback
(Link also in Chat)

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