



# Techniques for Solving Math Word Problems

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**[corelearn.com](http://corelearn.com)**



# Icons in the Session Today

Chat



- Enter response into Chat.



Pair Share

- You'll be working in a Pair-Share via breakout (sometimes a trio rather than a pair)

# Handouts for Today

- Pdf of problems you'll see today (now)
- Pdf of slides (at end of session)



# Noticing - Solving Word Problems

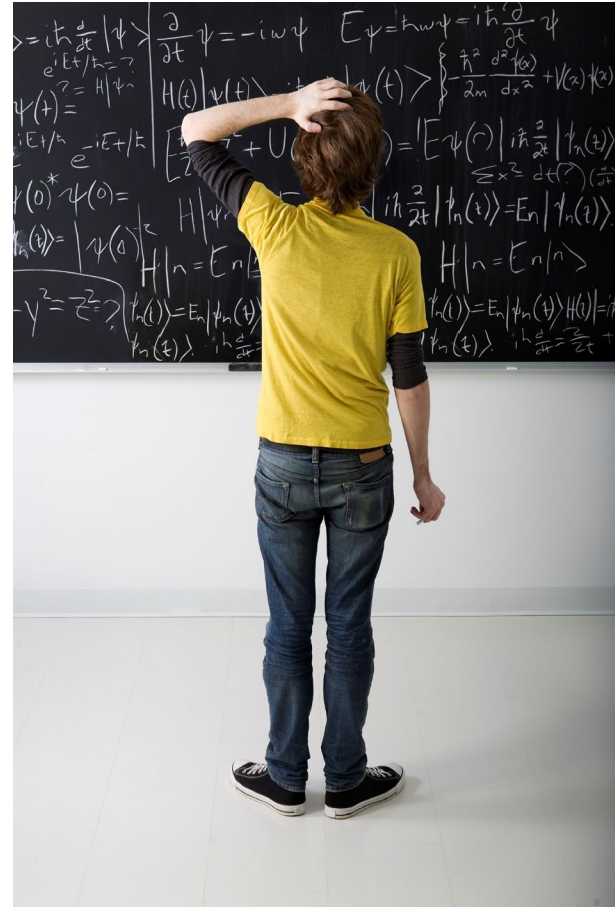


What important aspects of solving math word problems might this image of students working on a science experiment relate to?



# Wondering – Solving Word Problems

When you think of students solving math word problems what are the biggest challenges that come to mind?



# Objective for the Session

*Learn about and experience techniques for improving students' access to and success with math word problems.*

- Six categories of techniques
- 20+ techniques



# CCSSM Practice 4 – Model with Mathematics

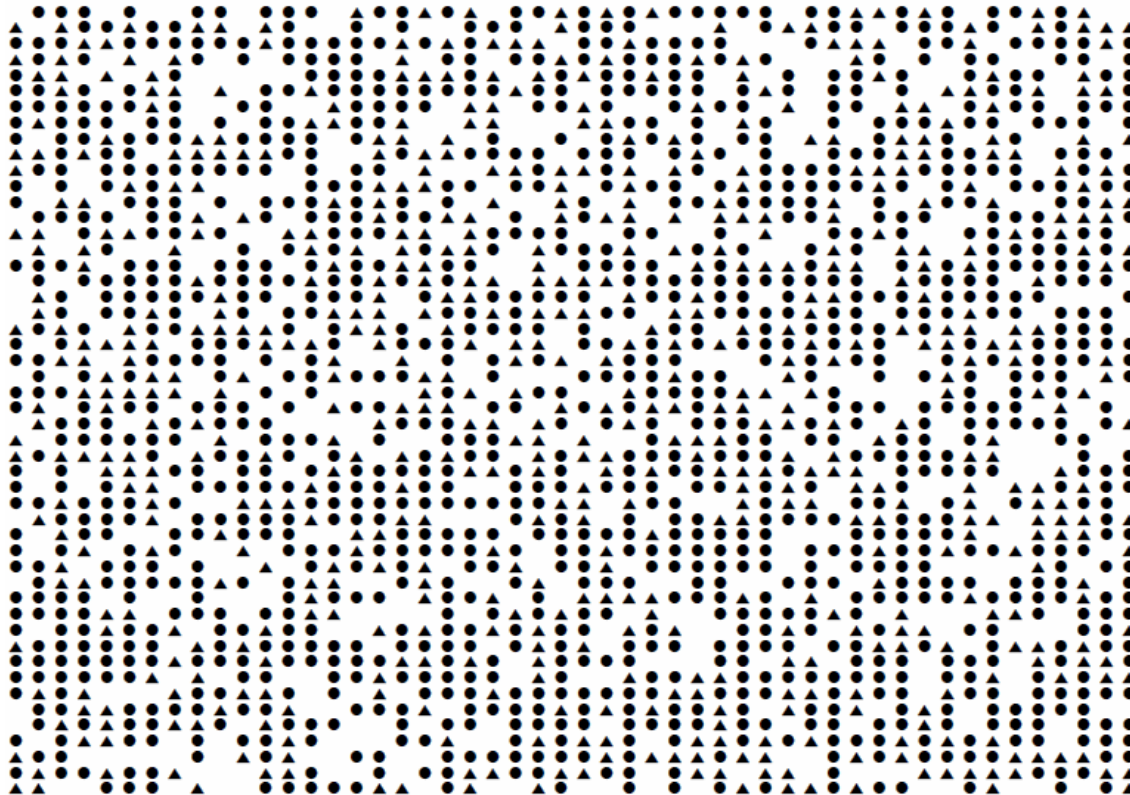
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

- In early grades, this might be as simple as writing an addition equation to describe a situation.
- In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
- By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

# MARS Task: Counting Trees

[www.map.mathshell.org/tasks](http://www.map.mathshell.org/tasks)

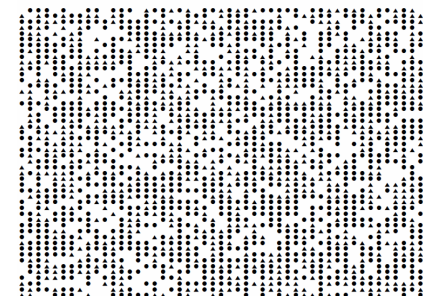
- This diagram shows some trees in a tree farm.
- Circles ● show old trees and triangles ▲ show young trees.





# Counting Trees

How many trees are there of each type?



50 by 50

- **Pre-Work:**

1. Estimate/guess the number of old trees and number of new trees (see diagram in handout). (30 seconds – write them down)
2. Share guess and reasoning with a partner.



Pair Share

- **Solve (5 min.):**

3. Discuss a plan for how to make a good estimate with a partner.
4. Solve using your plan without counting every tree (5 minutes).

**Old - 1275**  
**New - 658**

- **Post-Work:**

5. Compare solution and methods with other partners.
6. Stronger: share one idea about how they could make their explanation stronger.

# Techniques Used with *Counting Trees*

## ES – DS – CS

- **E**stimate/guess – **S**hare your reasoning
  - Estimate or guesstimate an answer, write this down, then share with someone else and explain your reasoning.
- **P**lan – **S**olve
  - Discuss the problem with a partner and plan how to solve it, then put your plan into action and solve the problem.
- **C**ompare– Make **S**tronger
  - Compare your answer and process with another pair of students and make one suggestion for how their explanation or process could be improved.

# George Polya or UPS Check

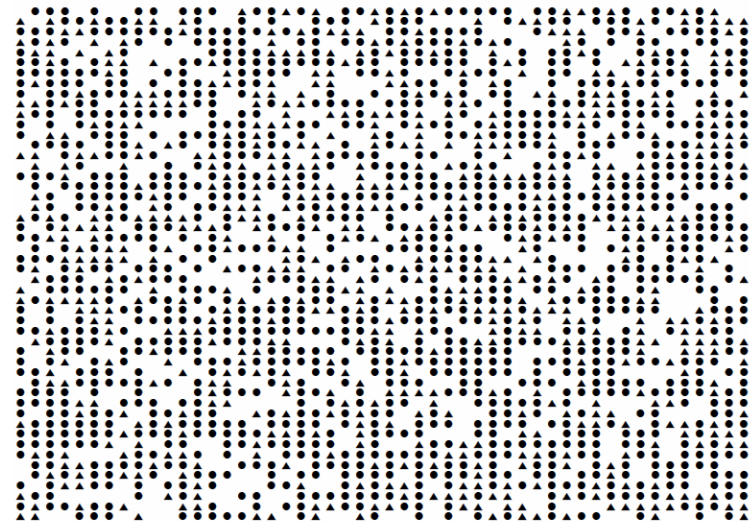
- **U**nderstand the problem
- **P**lan how to solve the problem
- **S**olve the problem
- **C**heck or look-back to answer and sensibleness of your answer

# MARS Task: Counting Trees

[www.map.mathshell.org/tasks](http://www.map.mathshell.org/tasks)

This diagram shows some trees in a tree farm. The circles ● show old trees and the triangles ▲ show young trees. Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

1. What method could he use to estimate the number of trees of each type? Explain your method fully.
2. On your worksheet, use your method to estimate the number of:
  - (a) Old trees
  - (b) Young trees



# Density of Text

*Mathematics is the most difficult content area material to read because there are more concepts per word, per sentence, and per paragraph than in any other subject; the mixture of words, numerals, letters, symbols, and graphics requires the reader to shift from one type of vocabulary to another.*

—Braselton & Decker, 1994

# IES Sample of Steps for Problem Solving

1. Identify the givens and goals of the problem.
2. Identify the problem type.
3. Recall similar problems to help solve the current problem.
4. Use a visual to represent and solve the problem.
5. Solve the problem.
6. Check the solution.

Institute of Education Sciences (IES), *Improving Mathematical Problem Solving in Grades 4–8*, 2012

# Problem Solving

Problem solving involves reasoning and analysis, argument construction, and the development of innovative strategies. These abilities are used not only in advanced mathematics topics—such as algebra, geometry, and calculus—but also throughout the entire mathematics curriculum beginning in kindergarten.

Institute of Education Sciences (IES), *Improving Mathematical Problem Solving in Grades 4–8*, 2012

# Six Categories of Techniques

- I. General Ideas
- II. Understand but Don't Yet Solve
- III. Using Visuals
- IV. Getting Student Buy-in
- V. Adapting Problems
- VI. Monitor, Process and Practice



# I. General Techniques

1. Polya's technique or *UPS Check*
2. **E**stimate-**S**hare / **P**lan – **S**olve / **C**ompare –  
make **S**tronger
3. Scaffold problems (chunk to focus on key  
information)
4. Teach and practice a routine  
(i.e., first read to understand; make initial estimates; collaborate; refine)

## II. Understand but Don't Yet Solve

5. Last sentence is the goal
6. Three Reads
7. Missing Number(s)
8. Parallel Problems
9. Represent visually but don't solve

# The Last Sentence is the Goal

Read or emphasize the last sentence first.

A rectangular container that has a length of 30 cm, a width of 20 cm, and a height of 24 cm is filled with water to a depth of 15 cm. When an additional 6.5 liters of water are poured into the container, some water overflows. **How many liters of water overflow the container?**

Use words, pictures, and numbers to explain your answer.  
(Remember:  $1 \text{ cm}^3 = 1 \text{ mL}$ .)

EngageNY/Eureka Math Grade 5 Module 5 End of Module  
Assessment #3

# Three Reads (Mathematical Language Routine #6) (Zwiers et al., 2017)

- 1) Students read the situation with the goal of comprehending the text (describe the situation without using numbers). ***[Get the gist]***
- 2) Students read the situation with the goal of analyzing the language used to present the mathematical structure. ***[Understand language]***
- 3) Students read the situation in order to brainstorm possible mathematical solution methods. ***[Plan]***

Understanding Language/Stanford Center for Assessment, Learning and Equity  
at Stanford University,. 2017

# Missing Number – Examples

## Pocket Money

Marcos has \$17 altogether. He has \$  in his hand and the rest of the money in his pocket. How much money does he have in his pocket?

Chat



## Cell Phone Deal

A phone originally sells for \$ . It is now on sale for  $\frac{1}{5}$  off the original price. April has a coupon for an extra 10% off the sale price. To the nearest dollar, how much less than the original price will April pay for the phone?

*from Ready Classroom Grade 7 Lesson 16*

# Parallel Problems

## Cell Phone Deal

What is the problem asking for? How might you solve the problem? (No credit for solving this version.)

A phone originally sells for \$500. It is now on sale for  $\frac{1}{5}$  off the original price. April has a coupon for an extra 10% off the sale price. To the nearest dollar, how much less than the original price will April pay for the phone?

*Adapted from Ready Classroom Grade 7 Lesson 16*

Solve this version of the problem

A phone originally sells for \$245. It is now on sale for  $\frac{1}{5}$  off the original price. April has a coupon for an extra 10% off the sale price. To the nearest dollar, how much less than the original price will April pay for the phone?

*from Ready Classroom Grade 7 Lesson 16*

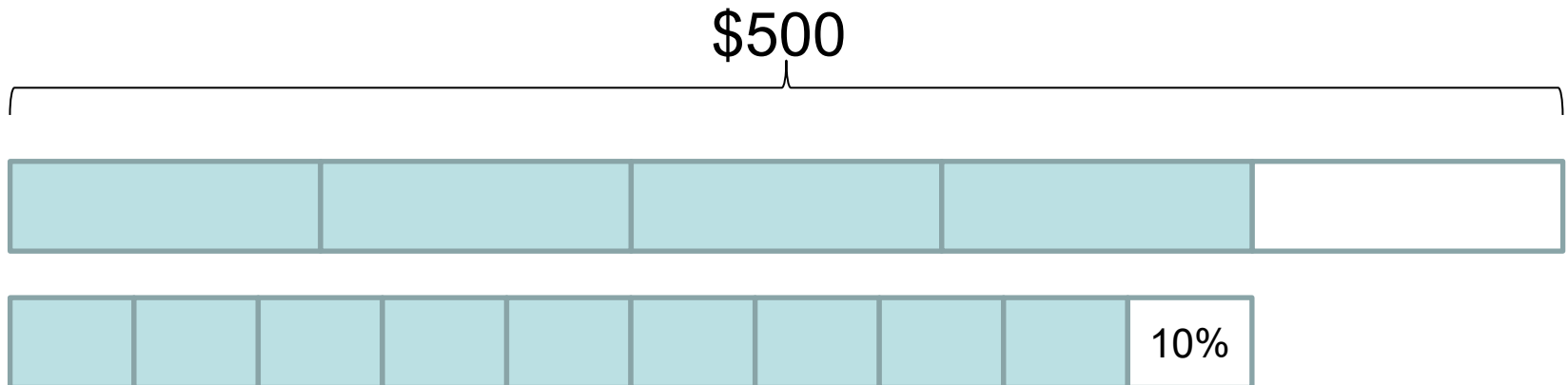
# Represent Visually but Don't Solve

## Cell Phone Deal

Create a tape diagram representing the problem situation.

A phone originally sells for \$500. It is now on sale for  $\frac{1}{5}$  off the original price. April has a coupon for an extra 10% off the sale price. To the nearest dollar, how much less than the original price will April pay for the phone?

*from Ready Classroom Grade 7 Lesson 16*



## III. Using Visuals

### 10. Diagramming, creating visual models

- *Tape diagrams*
- *Area models*
- *Number lines*
- *"Read/Draw/Write" (EngageNY/Eureka Math)*

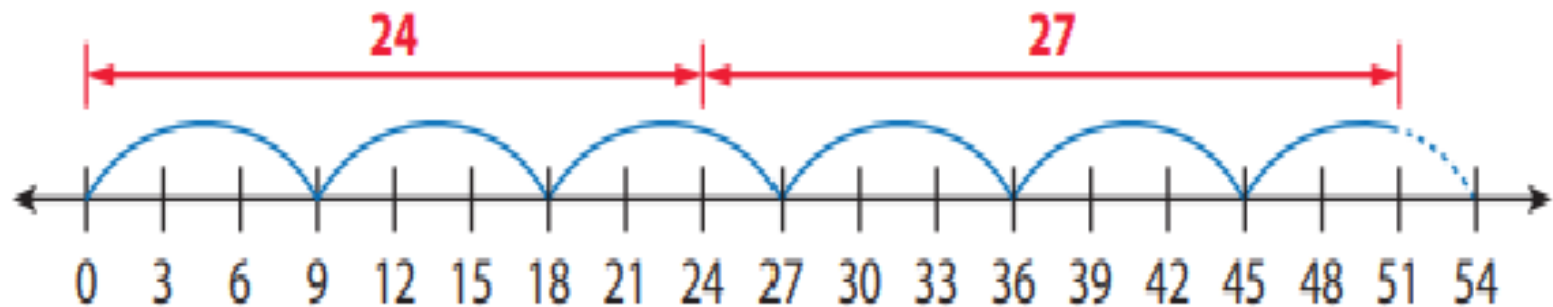
### 11. Matching models to problems

### 12. Matching expressions with terminology



# Visually with a Number Line

Ms. Dennison is packing up the books in her classroom for the summer. Each box holds 9 books. She has 24 math books and 27 science books to pack. How many boxes will she need?



Ready Classroom Grade 4 Lesson 10 p. 94 (2020)

# Read Draw Write (RDW)

*When faced with story problems young children will often arbitrarily add whatever numbers they see. The RDW process short-circuits this impulse by giving students tools to think about and model the relationships presented in a given situation. This enables them to solve problems accurately.*

*Lisa Watts-Lawton in Eureka Math blog at [greatminds.org/math/blog/eureka/post/problem-solving-the-rdw-way](https://greatminds.org/math/blog/eureka/post/problem-solving-the-rdw-way)*

# Solve in Desmos with Partner



Pair Share

Go to the [student.desmos.com](https://student.desmos.com) for the activity, *Initiating Access to Word Problems 1* and work in breakouts with a partner to solve the first five problems (slides 2-6).

Slide 7 is an extension/optional problem for those that want it.

<https://student.desmos.com/9csq32>

Hey, students!

Go to [student.desmos.com](https://student.desmos.com) and type in:

**9CS Q32**

You can also share this invitation link with your students:

<https://student.desmos.com/join/9csq32>

Copy

Have all students joined this class?

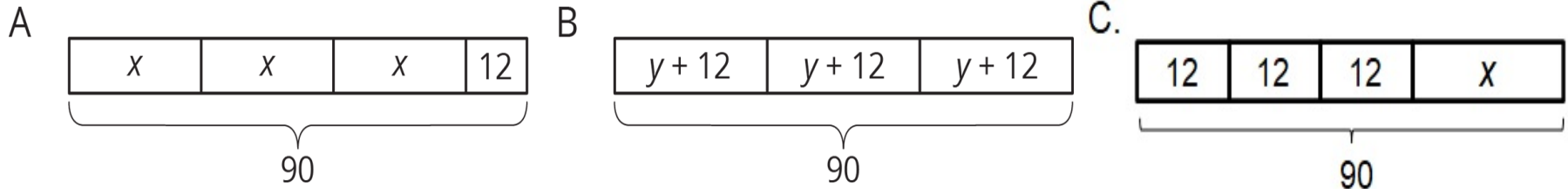
[Deactivate this invite code](#)

This prevents additional students from joining. You can always reactivate the code.

Do additional students need to join the class?

New students can use the code to join until **Oct 20, 2021**. [Change Date](#)

# Matching Models to Problems



Which diagram goes with which story? Be prepared to explain your reasoning.

**B Story 1:** Lin had 90 flyers to hang up around the school. First she gave 12 flyers to each of three volunteers. Then she took the remaining flyers and divided them up equally between the three volunteers.

**C Story 2:** Lin had 90 flyers to hang up around the school. After giving 12 flyers to each of three volunteers, how many did she have left to hang up by herself?

**A Story 3:** Lin had 90 flyers to hang up around the school. After giving the same number of flyers to each of three volunteers, she had 12 left to hang up by herself.

adapted from Open Up Resources (OUR) Grade 7 Unit 6 Lesson 6 Activity 3

## Match the word problem with the correct expressions at the bottom.

1. Write an expression to show a number increased by **11**.  $y + 11$
2. Write an expression to show a number decreased by **11**.  $y - 11$
3. Write an expression to show  $y$  less than **3.5**.  $3.5 - y$
4. Write an expression to show the sum of  $x$  and  $y$  reduced by **11**.  $(x + y) - 11$
5. Write an expression to show **5** less than  $y$ , plus  $x$ .  $y - 5 + x$

$$(x + y) - 11$$

$$y - 11$$

$$5 - y + x$$

$$3.5 - y$$

$$11 - (x + y)$$

$$y + 11$$

$$y - 5 + x$$

$$y - 3.5$$

Adapted from EngageNY/Eureka Math Grade 6 Module 4 Lesson 9 (2016)

# Matching Problems to an Expression

- 4 Which situation could be represented by the following expression? Circle all that apply.

$$48 + 2x$$

- ✓ A Sara's phone contract costs her \$48 per month, but she pays an additional \$2 for every minute she goes over her allotted minutes.
- ✓ B A fast food restaurant expects to use 48 eggs per day plus an additional 2 eggs for every customer coming in for breakfast.
- C A florist began the day with 48 roses and sold approximately 2 roses per hour.
- ✓ D Visitors to an amusement park pay an entrance fee of \$48 plus \$2 for each ticket purchased for the rides.

Ready Classroom Grade 6 Lesson 19 Practice p. 201 (2020)

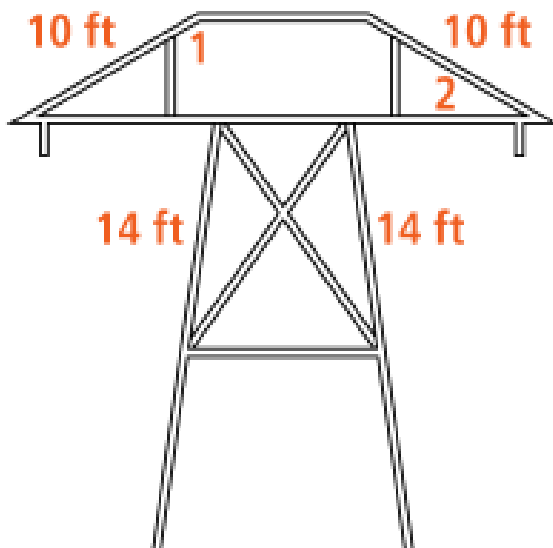
## IV. Getting Student Buy-In

13. Estimate/guess first and explain reasoning
14. Create word problems
15. Act it out
16. Vocabulary scavenger hunt
17. Could this be the answer? Why or why not?

# Could This Be the Answer?

All horizontal beams of the high-voltage transmission tower are parallel to the ground. The top section is an isosceles trapezoid. The center section is an isosceles trapezoid.

If the measure of angle 1 =  $128^\circ$  what is the measure of angle 2?



Could  $120^\circ$  be the answer?  
Why or why not?

Could  $10^\circ$  be the answer?  
Why or why not?

Could  $40^\circ$  be the answer?  
Why or why not?

Adapted from enVision Math Geometry Topic 6 Lesson 2 (2019)



# V. Adapting Problems

- 18. Differentiate – different numbers
- 19. Sentence frames

# Board the Roller Coaster

Jackson needs to be  $1 \frac{3}{4}$  inches taller in order to ride the roller coaster. Since he can't wait, he puts on a pair of boots that adds  $1 \frac{1}{5}$  inches to his height and slips an insole inside the boot that adds half as much as the boot does to his height. Will this make Jackson appear tall enough to ride the roller coaster?

Adapted from EngageNY/Eureka Math Grade 5 Module 3 (2016)

**What is the hardest part of this problem?**

Chat



# Scaffold to Increase Initial Access

(Three Reads, Missing Numbers, Friendlier Numbers, Diagrams)

## *Board the Roller Coaster*

Jackson needs to be  $1 \frac{3}{4}$  inches taller in order to ride the roller coaster. Since he can't wait, he puts on a pair of boots that adds  $1 \frac{1}{5}$  inches to his height and slips an insole inside the boot that adds half as much as the boot does to his height. Will this make Jackson appear tall enough to ride the roller coaster?

Adapted from EngageNY/Eureka Math Grade 5 Module 3 (2016)

**1. Get the Gist**

**2. Language &  
Quantities**

**3. Plan & Solve**

# Best Deal for Gina

*A store is having a sale – for everything over \$20 you get \$5 off the price. Gina has a \$12 off coupon for the same store. She can use either her coupon or the \$5 off sale, but not both. Gina is going to buy a \$20 shirt, \$35 shoes and \$10 socks. Should she use her coupon or go with the sale price?*

**What question do you think you will be asked about this problem?**

**Commit to ONE of the following without actually solving:**

- A. Coupon
- B. Sale price
- C. It doesn't matter. The cost is the same both ways.

# Best Deal for the Sweater

*One store is having a 50% off sale. Another store has a 40% discount, with an additional 15% off of the sale price. Which sale should you take advantage of if you want the best reduction on a sweater that costs \$68.79?*

**Commit to ONE of the following without actually solving:**

- A. The first store will give me the best price on the sweater.
- B. The second store will give me the best price on the sweater.
- C. It doesn't matter. The cost of the sweater will be the same in both stores.

# Turn and Talk with Sentence Frames

Partner share. Complete the following:

- *“What I first noticed about the problem was \_\_\_\_\_.”*
- *“I believe my answer makes sense because \_\_\_\_\_.”*

# Sentence Stems and Frames

- Specific sentence frames:
  - A. The store with the best reduction is \_\_\_ because \_\_\_\_.
  - B. The sales at the two stores are similar because they both \_\_\_\_.
  - C. The sales at the two stores are different because \_\_\_\_.
  - D. Comparing the sales at the two stores is tricky because \_\_\_\_.

# VI. Monitor, Process and Practice

- 20. Monitor and respond as needed
- 21. Select and purposely share
- 22. Practice and/or extend



# Pdf of Slides

- Dropping into Chat now.
- Also, putting in Chat link you can use to make a copy to your own google drive.



## Techniques for Solving Math Word Problems

Presenter

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CORE MATH

Thank you!

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**Feedback**  
(Link also in Chat)



  
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