

Purposeful Planning with Regard to Mathematical Progressions

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*“Be a first year teacher **FOREVER**”*

Success in secondary mathematics requires teachers’ skillful planning and deep understanding of mathematical concepts throughout formal schooling (K-12). With the goal of preparing students to achieve in algebra, Dr. Witzel will highlight progressions of learning mathematics through the challenges of algebra and shares the OPTIMIZE strategy for task analyzing curriculum.

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Set Your Goal

- Moore and Shulock (2010) found that nearly 70% of CA community college students do not graduate or transfer, most eventually dropping out.
- Students’ completion of a transfer level college math course within two years of enrollment was more highly correlated to completion than that of English courses.
- However, over one-half of community college students reported that Algebra was the highest math course they completed in High School making the average math placement by community colleges **two levels below a college level course**.
 - This means that when entering college, students are, on average, two math courses away from even receiving credit.
- In conclusion, students must succeed early in advanced level mathematics in high school.

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Mathematics Performance

A Barrier to Graduation

- Understanding of Algebra 2 is needed to complete Community College
- Students in college should focus on passing a math credit course within their first year

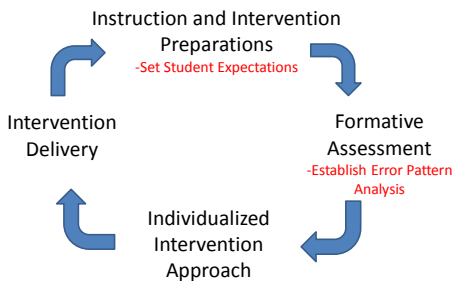
Translated to Real World Performance

- 78% of adults cannot explain how to compute interest paid on a loan
- 71% cannot calculate miles per gallon
- 58% cannot calculate a 10% tip

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Cycle of Instruction and Interventions



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Struggles in math per grade

- When entering your class for the first time, what do students not know that they should have learned in previous grades?

K
 1
 2
 3
 4
 5
 6
 7
 8
 Algebra
 Geometry
 Algebra 2

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Who is at fault?



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The college professor said, "Such wrong in the student is a shame,
Lack of preparation in high school is to blame."
Said the high school teacher, "Good heavens, that boy is a fool.
The fault, of course, is with the middle school."
The middle school teacher said, "From such stupidity may I be spared,
They send him to me so unprepared."
The elementary teacher said, "The kindergartners are block-heads all.
They call it preparation; why, it's worse than none at all."
The kindergarten teacher said, "Such lack of training never did I see,
What kind of mother must that woman be."
The mother said, "Poor helpless child, he's not to blame
For you see, his father's folks are all the same."
Said the father, at the end of the line,
"I doubt the rascal's even mine!"

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Program Goals

Program Goals/Objectives

1. Are the "**Big Ideas**" in the program obvious?
2. Do the "**Big Ideas**" match the Kansas College and Career Ready Standards?

A Big Idea is:

- Predictive of concept and skill acquisition and later math achievement (i.e., place value, equivalence)
- Something we can teach and improves student outcomes (i.e. achievement) when we teach it
- A significant amount of instructional time should be devoted to the development, relationships, and practice of these big ideas

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Instructional Preparations: OPTIMIZE

(Witzel & Riccomini, 2005; Riccomini & Witzel, 2011)

O	Order the math skills of a textbook chapter before teaching
P	Pair your sequence with that of the textbook
T	Take note of the similarities and differences
I	Inspect earlier chapters to see if they cover the differences. Check later chapters to see if they cover differences
M	Match supplemental guides to see if they cover the differences
I	Identify additional instruction to complement the current text
Z	Zero in the optimal sequence with your new knowledge
E	Evaluate and improve the sequence every year

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ORDER the math skills of a textbook chapter before teaching

Purpose: organize instructional sequence into perceived ideal sequence (most efficient & effective)

Steps

1. Examine first lesson and compare to last lesson
2. List concepts and skills required to achieve objectives

Look for:

- Coherence of critical concepts and component skills
- Include all necessary component skills to achieve objectives

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PAIR your Sequence with that of the Textbook

Purpose: Systematic and critical comparison of the two sequences

- Your ORDER and the publisher's sequence

Steps

1. List the sequence in the textbook
2. Side-by-side pairing and review/comparison of the two sequences
3. Identify similarities and differences
4. Identify potential areas requiring adaptations and modifications

Look for:

- Planned overlap of most important skills and concepts
- Sufficient time allocated to instruction and practice
- Large gaps or missing component skills

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TAKE note of the Similarities and Differences

Purpose: To identify similarities and differences in the two sequences to help plan and develop modifications and adaptations

Steps

1. Identify similarities and differences
2. Identify potential areas requiring adaptations and modifications

Look for:

- potentially problematic tasks for some students
- Target the concepts and skills that might be most difficult for students and require significant modification

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INSPECT and CHECK earlier and later chapters for coverage

Purpose:

1. Inspect earlier chapters to see if they cover the differences.
2. Check later chapters to see if they cover differences

Look for:

- Concepts and skills that are not addressed in earlier chapters
- For example: Previous chapters: $1/4 + 1/2$
Current chapter: $2/97 + 31/17$

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MATCH supplements and other available curriculum

Purpose: To examine the program's supplemental guides to see if they cover the differences

Steps

1. Following the publisher's directions, find the appropriate supplement and compare concepts and skills covered in supplements (e.g., Extra Practice, Enrichment, Remediation, etc)

Look for:

- Important component skills and relationships between Big Ideas
- Availability of supplements

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IDENTIFY additional instructional ideas

Purpose: To identify additional instruction to complement textbook (i.e., DIFFERENTIATE instruction for those students who might struggle)

Steps

1. Following the publisher's directions, find the appropriate supplement and compare concepts and skills covered in supplements (e.g., Extra Practice, Enrichment, Remediation, etc)
2. Design instruction following components of effective instruction

Look for:

- Important component skills and relationships between Big Ideas
- Explicitness of "differentiated" instruction

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ZERO in the optimal sequence..

Purpose: To merge best of both sequences to form most efficient and effective teaching sequence.

Steps

1. Rely on your content expertise in math and your experience with learners characteristics.
2. This is not an exact science, so give it a try!!!!
3. Consult relevant scientific research available

Look for:

- Student's types of errors or specific areas of difficulty

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Evaluate and Improve often

Purpose: To continuously evaluate and improve sequence every year

Steps

1. Monitor student progress towards unit goals and objectives
2. Evaluate students' achievement on high stakes assessment
3. Review students' performance and analyze for error patterns and deficit areas within and across the Big Ideas
4. Formulate plan to improve sequence for the next time

Look for:

- A better organized instructional sequence that is more beneficial to ALL students

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Example from a Math Department: Final sequence

- Begin charting your task analysis and how it matches the textbook.
- Example, simplifying expressions
 1. Addition and subtraction (preliminary skills)
 2. Addition with signed numbers (teacher includes)
 3. Distributive property (preliminary skills but with some revisiting)
 4. Recognize like variables v. numerals (included)
 5. Two variables with one coefficient (teacher includes)
 6. Two variables with multiple coefficients and distribution (included)
 7. Three variables with multiple coefficients and distribution (included)
 8. Over three variables with multiple coefficients and distribution (included)

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OPTIMIZE chart

Assumed sequence	Textbook sequence match	Resources	Final sequence
1. Check fluency of basic facts with signed numbers Relevance through actual equations by substitution	1. Solving equations using subtraction and addition	Previous textbook had a slower pace	1. Check fluency of basic facts with signed numbers
2. Equations by subtraction (include actual formulas)	2. Solving equations using multiplication and division	Multisensory Algebra Guide showed other ways to teach fractions	2. Relevance through supplementary guide activity
3. Equations by addition (include actual formulas)	3. Solving multi-step equations		3. Equations by subtraction (include actual formulas)
4. Equations by division (coefficients) (include actual formulas)	4. Solving equations with variables on both sides	Textbook from Previous Grade included some of the precursor skills her still students lacked	4. Equations by addition (include actual formulas)
5. Equations by multiplication (include actual formulas)	5. Multiple calculations of the variable on both sides		5. Equations by division (coefficients) (include actual formulas)
6. Equations with coefficients and addition or subtraction (include actual formulas)	6. Solving decimal equations		6. Teach fractions (reducing and multiplying)
7. Equations with coefficients and addition or subtraction (include actual formulas)	7. Actual formulas with variables including fractions	Algebraic Thinking for relevance discussion	7. Equations by multiplication (fractional coefficients) (include actual formulas)
8. Unknowns on both sides of the equal sign (include actual formulas)	8. Ratios and rate equations		8. Equations with coefficients and addition or subtraction (include actual formulas)
9. Decimal equations	9. Percent equations		9. Unknowns on both sides of the equal sign (include actual formulas)
10. Substitution with decimal equations			10. Decimal equations
			11. Substitution with equations

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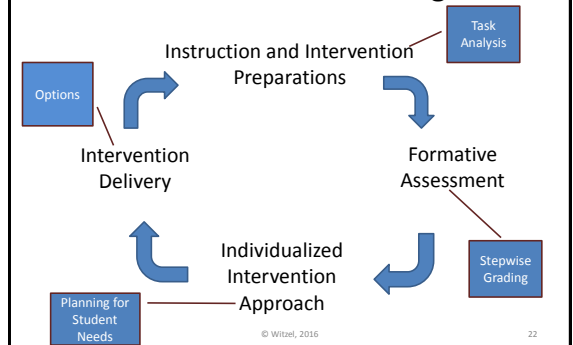
Reorganize a chapter (unit) in your textbook

Initial sequence	Textbook sequence	Resources	Final sequence

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Prelude to Future Strategies



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Vertical Alignment or Content Scaffolding

- Name a content scope and sequence that must be done correctly in order to lead to future success.
- What are some common mistakes when teaching this content?

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Base 10 Scope And Sequence

0	10	20	3
1	11	21	8
2	12	22	13
3	13	23	31
4	14	24	45
5	15	25	
6	16	26	
7	17	27	
8	18	28	
9	19	29	

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Place Value skills

- $27 = 2$ tens and 7 ones
- $45 = 4$ tens and 5 ones

Hundreds	Tens	Ones

- Should be represented physically and verbally
- Advanced learners should use place value within a calculation exercise.

Ones	Tenths	Hundredths

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Use place value to show long division: 6 grade CCSSM

$$\begin{array}{r} 250 \text{ remainder of } 1 \\ 3 \overline{)751} \\ \underline{6} \\ 15 \\ \underline{15} \\ 1 \end{array}$$

What is difficult about long division?

Rearrange 751 to work with multiples of 3
 $600 + 151 = 600 + 90 + 61 = 600 + 90 + 60 + 1$

$$\frac{600}{3} + \frac{90}{3} + \frac{60}{3} + \frac{1}{3} = 200 + 30 + 20 + \frac{1}{3} = 250 \frac{1}{3}$$

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Base 10 and Place Value

Solve $84 - 27$ in your head

Solve $84 - 27$ on this paper.

Write down the steps you used to solve it. Be honest.

What are the steps you used to solve it?

- a)
- b)
- c)
- d)
- e)
- f)

- a)
- b)
- c)
- d)
- e)
- f)

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Use Language Experiences

Base ten language

Tens	Ones
	■ ■ ■ ■ ■

Does this translate to decimals? How?

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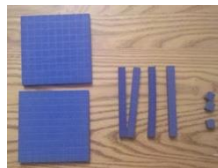
When you really know place value...

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How many hundreds are in this number?

How many tens are in this number?

How many ones are in this number?



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Use place value to add within 100

$$26 + 18$$

$$\begin{array}{r} 20 + 6 \\ + 10 + 8 \\ \hline 30 + 14 \end{array}$$

$$40 + 4 = 44$$

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Subtract within 100 using models and strategies based on place value

$$33 - 18$$

$$\begin{array}{r} + 30 \qquad + 3 \\ - 10 \qquad - 8 \\ \hline 20 \qquad -5 \end{array}$$

Reorganize (regroup/borrow)

$$\begin{array}{r} + 20 \qquad + 13 \\ - 10 \qquad - 8 \\ \hline + 10 \qquad + 5 \end{array}$$

$$10 + 5 = 15$$

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Place Value Progressions:

Make an array to show 24×76

- Fourth grade "Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models."

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- Fourth grade "Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models."
- $24 \times 76 = ?$

multiply	70	6
20	1400	120
4	280	24

$$1400 + 120 + 280 + 24 = 1824$$

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Place Value Progressions:

Make an array to show $2 \times \frac{3}{4}$

- Fourth grade "Apply and extend previous understandings of multiplication to multiply a fraction by a whole number."

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- Fourth grade "Apply and extend previous understandings of multiplication to multiply a fraction by a whole number."
- $(2) (\frac{3}{4})$

multiply	$\frac{3}{4}$ ths
2	$\frac{6}{4}$ ths

$$(2 \times 3) / 4 = 6/4$$

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Place Value Progressions:

Make an array to show $(2 \frac{1}{3})(4 \frac{1}{2})$

- Fifth grade "Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas."

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- Fifth grade "Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas."
- $(2 \frac{1}{3})(4 \frac{1}{2})$

multiply	2	1/3
4	8	4/3
1/2	2/2	1/6

$$8 + \frac{4}{3} + 1 + \frac{1}{6} = 8 + \frac{8}{6} + 1 + \frac{1}{6} = 10 \frac{3}{6}$$

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Place Value Progressions:

Make an array to show 7.6×2.4

Fifth grade "Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used."

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- Fifth grade "Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used."
- $7.6 \times 2.4 = ?$

multiply	7	.6
2	14	1.2
.4	2.8	.24

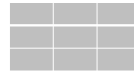
$$14 + 1.2 + 2.8 + 0.24 = 18.24$$

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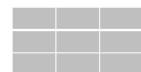
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Your Turn: How Arrays Translate

$$3 \frac{1}{4} \times 1 \frac{2}{3}$$



$$5.3 \times 2.4$$



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Answer: How Arrays Translate

$$3 \frac{1}{4} \times 1 \frac{2}{3}$$

x	3	1/4
1	3	1/4
2/3	6/3	2/12

$$\frac{3}{1} + \frac{6}{3} + \frac{1}{4} + \frac{2}{12} =$$

$$\frac{36}{12} + \frac{24}{12} + \frac{3}{12} + \frac{2}{12} = \frac{65}{12}$$

$$= \frac{65}{12} = 5 \frac{5}{12}$$

$$5.3 \times 2.4$$

x	5	0.3
2	10	0.6
0.4	2.0	0.12

$$10 + 0.6 + 2.0 + 0.12 = 12.72$$

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Place Value Progressions:

Make an array to show $(3x - 1)(4x + 5)$

Algebra "Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials."

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- Algebra "Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials."
- $(3x - 1)(4x + 5)$

multiply	$3x$	-1
$4x$	$12x^2$	$-4x$
$+5$	$15x$	-5

$$12x^2 - 4x + 15x - 5$$

$$12x^2 + 11x - 5$$

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$$(2x^2 - x + 6)(7x - 9)$$

multiply	$2x^2$	$-1x$	$+6$
$7x$	$14x^3$	$-7x^2$	$42x$
-9	$-18x^2$	$+9x$	-54

$$14x^3 - 25x^2 + 51x - 54$$

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Construct Viable Arguments and Critique the Reasoning of Others

- Set expectations of mastery
- What to look for in student work

Students talk about and ask questions about each other's thinking in order to clarify or improve their own mathematical understanding	1	2	3	4
	Strongly Disagree			Strongly Agree

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Chinese v. US Math Text Analysis

(Cheng & Wang, 2012)

- An earlier curriculum with a strong focus on number sense development presumably helps children perform better in mathematics later on.
- Chinese students outperformed their United States (U.S.) peers on number sense at lower grade levels, and on many other mathematics areas at both lower and higher-grade levels.
- The two countries textbooks differ as well.
- They found that Chinese textbooks focused more on the meaning and representation of number, place value, base-ten concepts, and on the connection of number sense to number operation.
- U.S. textbooks focused more on number counting, patterns, and the connection of number sense to data analysis.

Don't let textbooks become a curriculum!

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Mathematics Curriculum

"The single most important factor in predicting whether or not a teacher will be effective is whether the curriculum that is delivered to students in his or her classroom is linked logically or empirically to the outcomes that are desired."

(Berliner, 1986p. 128-129).

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Key Curricular Resource

(adapted from Riccomini & Witzel, 2011)

- Math textbooks influence a majority of classroom activities
 - Especially for new teachers and teachers who have not been well prepared to teach mathematics
 - Teachers with poor math content expertise
 - Lack of professional development in the area of mathematics (Malzahn, 2000)
- Textbooks are the primary means of presenting new content

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Textbooks Make a Difference

- Curriculum design influences student performance (e.g., Woodward, Baxter, & Robinson, 1999).
 - Students receiving the more explicit approach significantly outperformed the students instructed with *Everyday Mathematics*
- Mathematica Policy Research (Agodini, et al., 2009) analyzed four elementary textbooks. Findings:
 - Achievement was significantly higher in schools assigned to Math Expressions and Saxon Math than in schools assigned to Investigations and Scott Foresman-Addison Wesley Mathematics.
 - Better performing programs led to higher achievement for several student subgroups, including students in schools with low math scores and students in schools with high poverty levels.
- Modifications to mathematics texts are essential for students who are struggling

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Selecting Mathematics Textbooks

Needs within Math Textbook Adoption

- **Match** the learning needs of students with that of the textbook approach
- Provide intervention and acceleration **options**
- **Balance** conceptual knowledge with procedural facility
- Design **accessibility** into instruction to meet the needs of low performing students and students with disabilities

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Your Textbooks

- Does your district have a textbook, key resource, or none?
- If so, name strengths and weaknesses.
- Do the topics align?
- Do the strategies align?

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Evaluating Your Key Resource(s)

- Reviewing and modifying math curricula is **essential**, but not necessarily **sufficient** to produce improved mathematical performance.
- We also need well-managed classrooms and proper implementation (**instructional delivery**) of well-designed programs

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Vertical Alignment of Strategies

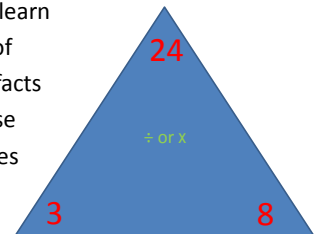
- Is it important for students to be familiar with certain approaches or strategies across multiple grade levels? Why or why not?
- Have you ever taught a lesson based on what you thought was common only to find it was unfamiliar to the students?
- Name some helpful approaches:
 - Number lines
 - Arrays
 - ????

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Single Strategy Scaffolds: Fact Triangles

- Students must learn combinations of multiplication facts
- Organizing these facts by triangles sets up future strategies

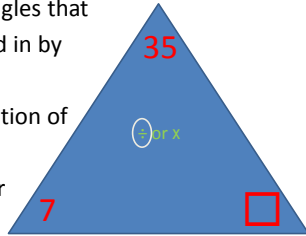


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Fact Triangles

- Set up fact triangles that need to be filled in by the student
- Scaffold completion of the triangles as students master the facts

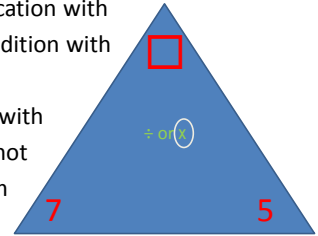


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Fact Triangles

- Match multiplication with division and addition with subtraction
- Triangles work with number lines, not instead of them

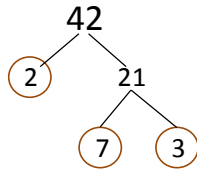


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Fact Triangles Turn into

- Factor trees for prime and composite numbers
- Have students denote the prime numbers



Example

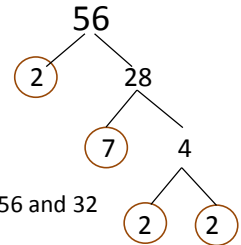
- $42 = (2)(7)(3)$
- The prime factors of 42 are 2, 7, and 3

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Factor Trees

- Factor trees help explain Greatest Common Factors (GCF)



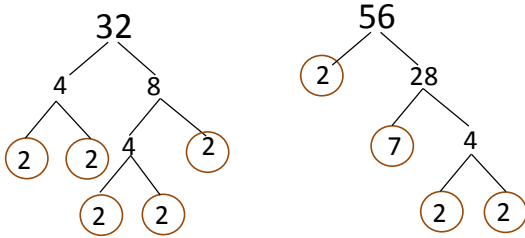
Let's Try finding the GCF of 56 and 32

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Factor Trees

- Find the GCF and LCM of 32 and 56



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Factor Trees



- Prime Factors of 32 are $2 \times 2 \times 2 \times 2 \times 2$
- Prime Factors of 56 are $2 \times 7 \times 2 \times 2$

Common Prime Factors are:

$2 \times 2 \times 2 \times 2 \times 2$

$2 \times 7 \times 2 \times 2$

GCF = $2 \times 2 \times 2 = 8$

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Factor Trees

- Factor trees help also help show the Least Common Multiple (LCM)



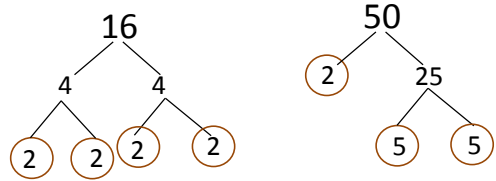
The GCF was 8, made up of $2 \times 2 \times 2$
 Use the remaining prime factors to compute the least common multiple. $LCM = GCF \times 2 \times 2 \times 7$
 $LCM = 8 \times 2 \times 2 \times 7$

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Factor Trees

- Find the GCF and LCM of 16 and 50



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Factor Trees



- Prime Factors of 16 are $2 \times 2 \times 2 \times 2$
- Prime Factors of 50 are $2 \times 5 \times 5$

$GCF = 2$

$LCM = 2 \times 2 \times 2 \times 2 \times 5 \times 5$

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Next Steps

- Have grade level team horizontally and vertically map their approaches
- Evaluate the alignment of your curriculum
 - Content
 - Strategies
- Revise the alignment to ensure core programs scaffold key features (content and strategies)
- Determine for fidelity of teachers' instruction



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Summary

- In what math areas do students struggle the most?
- What is OPTIMIZE and why might it be important?
- Vertical Planning
 - Name a content scaffold
 - Name a strategy scaffold

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"The most incomprehensible thing about mathematics is that it is actually quite comprehensible" (Kiran Mali)

Questions?



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