KANSAS
MULTI-TIER SYSTEMS OF SUPPORTS (MTSS)
&
ALIGNMENT

MATH GUIDE

PRE-K THROUGH 12TH GRADE

Introduction to Document

The *Kansas Multi-Tier System of Supports (MTSS) and Alignment PK-12 Math Guide* has been created to assist teams in documenting and utilizing the structures necessary to implement the Kansas MTSS and Alignment framework. This guide also provides steps to support districts in successfully completing the tasks and decision making necessary for a sustainable system. Content-area-specific guides for reading and behavior and social-emotional learning are companion documents to this one, providing information specific to each respective content. All Kansas MTSS and Alignment documents are aligned with the Kansas MTSS: Innovation Configuration Matrix (ICM), which describes the critical components of a Kansas MTSS and Alignment framework and what each component looks like when fully implemented, and the Kansas Multi-Tier System of Supports: Research Base, which provides a basic overview of the research support for the Kansas MTSS and Alignment.

www.ksdetasn.org/mtss

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Introduction

In Kansas, there is a belief that all children can learn. Fundamentally, every student should be challenged to achieve high standards, both academically and behaviorally. A systemic framework for ensuring that all students have this experience is referred to as the Kansas Multi-Tier System of Supports and Alignment (MTSS). Simply put, the Kansas MTSS and Alignment framework is a set of evidence-based practices implemented across a system to meet the needs of all learners. Horner et al. (2005) stressed the importance of supporting children both academically and behaviorally in order to enable them to reach their fullest learning potential. The Kansas MTSS and Alignment framework builds a system of prevention, early intervention, and support to ensure that all children learn. Additionally, Kansas MTSS and Alignment establishes a system that intentionally focuses on leadership, professional development, and an empowering culture in addition to a focus on student learning.

The Kansas MTSS and Alignment framework incorporates a continuum of assessment, curriculum, and instruction. This systemic approach supports both struggling and advanced learners through the selection and implementation of increasingly intense evidence-based interventions in response to both academic and behavioral needs. Whether a district's program is implementing a single content or planning to integrate both academic and behavior content, it is essential to begin with the Phase 1 Guide and then the content guides. The Kansas MTSS and Alignment framework establishes a self- correcting feedback loop that includes ongoing monitoring of the effectiveness of instruction to ensure that each Kansas student achieves high standards.

Across the nation, schools use a variety of curricula, interventions, and methods to monitor student learning, both academically and socially. The goal of Kansas MTSS and Alignment is to provide an integrated systemic approach to meet the needs of all students. To achieve this, resources must be used effectively and efficiently. While the Kansas MTSS and Alignment framework does not necessarily require additional resources or making additions to existing practices, it does involve evaluating current practices to identify those that yield evidence of effectiveness, addressing areas that are missing, and replacing ineffective or inefficient approaches with those that are supported by research and/or evidence. The Kansas MTSS and Alignment is a guiding framework for school improvement and accreditation activities to address the academic and behavioral achievement of all students.

Structuring the Kansas MTSS & Alignment Math Framework

Demand for Mathematics

Simply put, math matters. Many in our society have deemed it acceptable to be "bad at math," while on the other hand, one would be hesitant to confess their inability to read. The National Research Council (NRC) declares, "For people to participate fully in society, they must know basic mathematics. Citizens who cannot reason mathematically are cut off from whole realms of human endeavor. Innumeracy deprives them not only of opportunity but also of competence in everyday tasks" (NRC, 2001, p.1). Math principles are not only ingrained in everyday tasks, such as shopping for groceries, creating a monthly budget, and comparing interest rates, but the demand for math proficiency also correlates to current workforce opportunities. According to *U.S. News & World Report* (2016), the employment rate for jobs in the STEM field since 2000 has increased at a 22% higher rate than all other jobs. A similar report from 2015 estimated that jobs in which qualifications extend beyond a high school diploma, but not a four-year degree, require solid math skills and a strong level of digital proficiency. It is apparent that a call for proficiency in mathematics is required.

How are students in the United States measuring up according to the National Center for Educational Statistics (2021)? The Trends in International Mathematics and Science Study (TIMSS) assesses fourth and eighth grade students in number, measurement and geometry, data, and algebra (eighth grade only) and is conducted every four years. In 2019, fewer than 50% of fourth graders and fewer than 40% of eighth graders met the level of "high standards" on this assessment, demonstrating no real change from the 2015 study. The Programme for International Student Assessment (PISA) measures 15-year-olds' ability to use their reading, mathematics, and science knowledge and skills to meet real-life challenges. In math, the U.S. ranks 36th out of the 79 countries and regions that participated. U.S. math performance has not changed much since the first PISA tests in 2000, despite many well-intentioned efforts and initiatives by educators.

According to *Trajectories of Mathematics Performance: From Preschool to Postsecondary* (Powell & Nelson, 2016), student math performance in the elementary grades, such as skills with counting, numbers, calculation, and quantity comparison, reliably predict math achievement in later grades. In addition, skills in number line estimation and computation predict proficiency with fractions; fraction proficiency predicts math performance in middle and high school. Finally, the authors maintain that math ability in high school is directly correlated to hourly earnings upon entering the workforce (Powell & Nelson, 2016). Mathematical concepts build upon each other year after year, and unless students are given the proper instructional support early on, they are set on a path that leads to decreased postsecondary opportunities. It is critical that educators address this sense of urgency and take the necessary steps to improve mathematics instruction for all students.

Curriculum

Critical Concepts in Mathematics

A discussion of critical concepts in math is important to ensure that personal experiences and feelings about math do not interfere with the ability to hold high expectations for student achievement in math. Teacher expectations for student achievement and other beliefs about math are often communicated to students in very subtle ways.

John Hattie's *Visible Learning* effect sizes are documented on the [MetaX website.](#) The research reflects that teachers' estimates of student achievement yield an incredibly high effect size (1.29) on student learning. Therefore, a discussion of the critical concepts in math can help ensure that teachers understand some of the basic issues in the field and provide mutual support for developing productive practices for student learning and math instruction.

The following five critical concepts for math for school-age students were adapted from Riccomini and Witzel (2010), and the source for each critical concept is also referenced below:

- All students can be mathematically proficient (Kilpatrick, Swafford, & Findell, 2009).
- All students need a high-quality mathematics program (National Mathematics Advisory Panel, 2008).
- Effective mathematics programs must teach conceptual understanding, computational fluency, factual knowledge, and problem-solving skills (National Mathematics Advisory Panel, 2008).
- Effective mathematics instruction matters and significantly impacts student learning (National Mathematics Advisory Panel, 2008; Newman-Gonchar, Clarke, & Gersten et al., 2009).
- Teachers should use a balance of student-centered and teacher-centered instruction in the core mathematics program (National Mathematics Advisory Panel, 2008).

Tier 1 Mathematics

Math Proficiency

What does it mean for students to be mathematically proficient? In the book *Adding It Up: Helping Children Learn Mathematics* (NRC, 2001, p. 5), mathematical proficiency is the terminology used to convey the development of success in mathematics. The report states that mathematical proficiency has five strands, and "the most important feature of mathematical proficiency is that these five strands are interwoven and interdependent." The five intertwined strands of mathematical proficiency are as follows:

- **Conceptual Understanding**: Comprehending mathematical concepts, operations, and relations – in other words, knowing what mathematical symbols, diagrams, and procedures mean
- **Procedural Fluency**: Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately
- **Strategic Competence**: The ability to formulate problems mathematically and devise

strategies for solving them using concepts and procedures appropriately

- **Adaptive Reasoning**: Using logic to explain and justify a solution to a problem or to extend from something known to something not yet known
- **Productive Disposition**: Seeing mathematics as sensible, useful, and doable – if you work at it – and being willing to do the work

It is necessary that students be given frequent and equitable opportunities to strengthen each proficiency strand within their mathematics classroom.

Standards for Mathematical Practice

Mathematical proficiency has been proven repeatedly in the research to be one of the greatest predictors of future success in students. Early mathematics competence is evidenced to be one of the best predictors of school success across the curriculum (Duncan et al., 2007; NAEYC/NCTM, 2010).

The Standards for Mathematical Practice (SMP) describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on key processes and proficiencies with longstanding importance in mathematics education. The eight SMP are listed as follows (Common Core, n.d.):

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

The KSDE Mathematics Flip Books define each of these practices as well as provide examples of how these might be applied at each grade level. See 1st grade examples here.

Essentials for Core Mathematics

Essential core mathematics instruction, K-12, should occur for a minimum of 50 to 60 minutes per day (National Council of Teachers of Mathematics, 2006; Riccomini & Witzel, 2010). The math core should be effective with the majority of students and include differentiated instruction for students who experience math difficulty despite meeting the benchmark standard. The following are essential components of core mathematics instruction:

- The ingrained belief that all children *can* achieve proficiency in mathematics.
- A high-quality program for each individual child.
- Instruction that addresses the five strands of mathematical proficiency.

- Effective instruction embedded within all instructional practices at every tier.

Mathematics instruction in pre-kindergarten through eighth grade should be sequential and emphasize a well-defined set of critical topics. The math curriculum should not “revisit topics year after year without bringing them to closure” (National Mathematics Advisory Panel, 2008). In other words, students must learn critical skills at a level of conceptual understanding, proficiency, and fluency that enables automaticity in math computation and problem solving. Rather than racing to cover topics in a mile-wide, inch-deep fashion, the Kansas Standards require us to significantly narrow and deepen the way time and energy are spent in the math classroom. Achieve the Core illustrates the progression of these critical concepts from kindergarten through eighth grade in this chart.

The Kansas MTSS and Alignment Math team has adapted the Achieve the Core chart to portray the 2017 Kansas Math Standards so that educators can see the progression of skills and critical focus areas across grade levels in the PreK-8 Math Instructional Foci document.

In addition to the critical mathematical concepts at each grade level, consider the three shifts in mathematics identified by the Common Core mathematics standards when ensuring the vertical alignment of your PreK-12 mathematics program:

- **Focus** strongly where the standards focus
- **Coherence**: Think across grades and link to major topics within grades.
- **Rigor**: In major topics, pursue conceptual understanding, procedural skill and fluency, and application with equal intensity.

For further information on the shifts, see College- and Career-Ready Shifts in Mathematics.

Math Core Curriculum Considerations

Because the core curriculum is the comprehensive curricula that ***all*** students receive, materials that comprise the core curriculum must support good quality classroom instruction to ensure that all students meet or exceed the Kansas Standards.

In order to evaluate the core curriculum materials, staff members must:

- Analyze which materials are currently in use
- Examine their alignment with the Kansas Standards
- Look at the evidence regarding their effectiveness
- Determine if there is a need to strengthen the core curriculum

Research shows that the curriculum chosen for core instruction can make a difference in the achievement level attained by students. For this reason, it is important to review the available evidence regarding the effects of math curricula that might be under consideration. When selecting a core curriculum, look for alignment with the standards, critical areas that are taught both conceptually and procedurally, and a balance between student-centered and teacher-directed lessons. It is also recommended that educational leaders consider alignment of instructional practices and mathematics vocabulary between core and intervention resources

(Nelson, Pfannenstiel & Edmonds, 2019). Below are online resources that can be used to locate evidence regarding a specific math curriculum:

- EdReports
- What Works Clearinghouse
- Best Evidence Encyclopedia
- Evidence for ESSA

When considering a curriculum adoption for mathematics, read the Kansas Instructional Curriculum/Resource Adoption Process Guide for more resources and guidance.

Tier 2 and 3 Mathematics

MTSS Hybrid Model

When fully implementing MTSS, supplemental and intensive support is provided through a hybrid intervention model that combines a protocol and problem-solving approach to ensure a rapid response to meet student needs as they arise. The protocol aspect of the hybrid model requires that each building preselect a set of interventions that will be used, as student data indicate a need for support beyond the core. The problem-solving aspect of the MTSS hybrid model is used to further identify and customize supports for students, especially at the Tier 3 level.

Considerations for Math Intervention Materials

Students experiencing difficulty with whole-number knowledge most likely struggle with one or more of three central areas: early numeracy, computation, or word-problem solving (Codding et al., 2017). Gersten et al. (2009a) outlined in the *Institute of Education Sciences (IES) Practice Guide* that, when considering these mathematical concepts for intervention, educators can apply the following recommendations for intervention materials:

- Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade five and on rational numbers in grades four through eight. These materials should be selected by a committee.
- Intervention materials should include opportunities for students to work with visual representations of mathematical ideas, and interventionists should be proficient in the use of visual representations of mathematical ideas.

Evaluating Intervention Materials

Codding et al. (2017) shared eight features for evaluating current and/or potential math intervention materials. These eight features have been adapted into the following questions for math teams to use for guidance:

1. Is content provided using explicit instruction?
2. Is strategy instruction incorporated?

3. Is instruction sequenced logically?
4. Are progress monitoring and feedback embedded?
5. Are drill, practice, and cumulative review activities included?
6. Is student verbalization of the problem-solving process modeled and encouraged?
7. Are visual representations used?
8. Is reinforcement provided?

Just as the core curriculum was reviewed and evaluated by staff members, it is imperative that instructors review the current Tier 2 and Tier 3 materials to determine what will work best to meet the academic needs of all students. By conducting this review, staff members will be positioned to make the necessary decisions regarding whether there are gaps in materials that should be filled. Staff members will also make decisions about discontinuing or replacing curricula due to the lack of effectiveness or an evidence base.

For students in upper elementary and middle school, interventions should be more targeted to key algebra-readiness progressions. Interventions should, at a minimum, place heavy emphasis on the computation of whole numbers, working with fractions, and solving equations.

There are a wide range of needs during intervention at any grade level; therefore, it is vital that schools have interventions that comprehensively address all critical areas of mathematics and cover both conceptual and procedural aspects of mathematics. Often content-specific targeted interventions lack depth, focusing primarily on rogue procedures.

Websites for Evidence-Based Math Interventions and Strategies:

- National Center on Intensive Intervention
- Evidence-Based Intervention Network
- What Works Clearinghouse
- TeachingLD
- Evidence for ESSA

Instruction

Tier 1 Mathematics

It is recommended that core instruction create a balance between teacher-centered and student-centered instruction (National Mathematics Advisory Panel, 2008). During teacher-centered instruction, the teacher directly teaches concepts using explicit instruction. In student-centered instruction, the teacher guides students in constructing meaning through discovery learning. However, this approach is only appropriate once students have shown accuracy and fluency with the specific skills needed to successfully complete the task (VanDerHeyden & Codding, 2020; Buongiovanni, 2021). For students who are struggling, low achievers, and students with disabilities, teacher-centered instruction has been demonstrated to be especially important (Gersten et al., 2009b).

The Science of Math

Common Misconceptions

The Science of Math is a movement that utilizes objective evidence regarding how students learn math most effectively in order to make educational decisions in addition to informing policy and practice. The following table outlines several common misconceptions and the truths according to research. A wealth of resources can be downloaded and printed from the site linked above. The information contained in Table 1 details various misconceptions from the *Science of Math* alongside their evidence-based truths.

Table 1

Misconceptions versus truths according to the Science of Math.

Misconception	Truth
Procedural and Conceptual Understanding Some educators believe that students should not be exposed to procedural instruction until they have demonstrated adequate conceptual understanding of a topic.	Conceptual knowledge supports procedural knowledge AND procedural knowledge supports conceptual knowledge. They should be taught together!
Growth Mindset Many educators believe interventions targeting a growth mindset will improve academic achievement.	Intervention research on stand-alone growth mindset interventions yield minimal gains on GPA in mathematics courses, and replication attempts have failed. The most effective way to improve academic achievement is to deliver skill-building intervention.
Explicit Instruction Inquiry-based instruction should be the primary tactic used to teach students. Explicit instruction only is beneficial for struggling learners. Explicit instruction is an instructional tactic through which students are provided with correct answers and only promotes rote learning to passive learners.	Explicit instruction offers value through sequencing of tasks in increments of difficulty, fluency building that promotes effective practice, and scaffolded opportunities for students to combine learned skills with new knowledge. Explicit instruction facilitates creativity.
Productive Struggle Many educators believe that struggling or grappling with challenging math tasks causes students to gain a deeper	Productive struggle does not deepen understanding, grit, or creative problem solving. Productive struggle can lead to frustration and cause students to develop misconceptions. In

understanding than would be achieved if they learned the same skill without a struggle.	addition, the false starts that are involved in struggling with challenging tasks without adequate support or guidance lead to lost instructional time and inefficiency.
Algorithms Many educators believe algorithms promote memorization, and this would contribute to a superficial understanding of steps, conventions, and rules. This belief leads to the idea that students should not be taught algorithms.	An algorithm is a step-by-step procedure for solving a problem. Using an algorithm requires conceptual understanding of what is happening in the problem and procedural knowledge to accurately solve. Algorithms can serve as a link between conceptual understanding and procedural knowledge.
Math Anxiety Many educators believe math anxiety is caused by instructional activities and timed tests. In schools, educators may interpret students' disengagement in math activities or statements that they dislike math as math anxiety. Educators may reduce the difficulty of a math lesson or remove timed tests as a way to reduce math anxiety.	No studies have determined that timed tests cause math anxiety, defined as feelings of apprehension, tension, or fear that may interfere with performance on math-related tasks. In fact, timed tactics improve math performance.

Instructional Practices

Cognitive Load Theory

When evaluating instructional practices, considering the cognitive science regarding *how* students learn is essential. Cognitive Load Theory (CLT), a theory that distinguishes between types of learning and just how much new information one can learn at a time, ventures to explain why some particular instructional approaches result in more learning than others, and its observations are founded on evidence from experiments (Ashman, 2023). CLT divides the memory into two main categories: working memory (WM) and long-term memory (LTM). There is a limit to working memory (WM), but long-term memory (LTM) is unlimited. It is easy for chunks of knowledge to be lost if they're not transferred from WM to LTM within a reasonable amount of time. Short-term testing and quizzes are successful because of this. However, when students must recall the information again a few months later, they act as if they never learned it. Are you familiar with this scenario in your math classroom? Since mathematics is hierarchical, students must transfer critical mathematical concepts and procedures to LTM. Several instructional strategies ensure that learners will be able to make that transfer.

In order to discuss these instructional strategies effectively, we must first understand the difference between novice and expert learners. It is generally accepted that the classroom teacher is the expert and students are novices. It is important to remember that students are not mini-experts since they know less about that particular field. Their lack of prior knowledge around that specific domain causes them to think about and approach problems differently. "A teaching approach that works well with an expert will most probably not work well with a beginner and can even be detrimental to their learning" (Kirschner & Hendrick, 2020). For experts, discovery learning, problem-based learning, or inquiry learning are examples of instructional approaches that may be well suited because they assume the expert has a high level of knowledge and/or already knows how to solve the problem. This would suggest that an expert is far more likely to arrive at the correct solution. Novice learners, however, require a more structured approach involving modeling, worked examples, step-by-step explanations, and lots of opportunities for practice since they possess a lower level of knowledge. The number of pathways they may take toward a solution make it improbable, but not impossible, for them to eventually land on the correct solution (Ashman, 2021). In this regard, teachers would be more likely to accomplish their educational goals if they routinely introduced a direct and structured instructional approach to students (i.e., novice learners).

Regarding the mathematics that you teach, foundational skills are imperative for students to be equipped to retrieve those skills from LTM quickly and easily. As a result, they will be able to concentrate on learning the relevant mathematics concept of the day through their WM. When teaching long division, students who can easily recall basic multiplication and subtraction and comprehend place value can focus on long division steps without getting bogged down in prerequisite skills since the limited space in their WM can instead be used for learning the steps. Students can explore and enjoy math more easily when they can easily retrieve declarative facts and procedures (i.e., have automaticity). When students enjoy math and teachers enjoy teaching it, math becomes enjoyable!

Explicit Instruction

Explicit instruction is a non-negotiable instructional practice for *all* tiers of mathematics instruction. Archer and Hughes (2011) expressed, "One of the greatest tools available to us... is explicit instruction – instruction that is systematic, direct, engaging, and success oriented."

Visible Learning research shows that explicit teaching strategies have an effect size of 0.63, which equates to more than one year's growth for one year of input. We define explicit instruction as the inclusion of clear statements about the purpose and rationale for learning the new skill, clear explanations and demonstrations of the instructional target, and supported practice with feedback until independent mastery is achieved. Explicit teaching strategies involve teacher-led instruction with a series of supports or scaffolds throughout the learning process from modeling to guided practice to mastery (Adapted from Visible Learning MetaX).

Read our Explicit Instruction handout for more details about the true implementation of the many components of explicit instruction.

Systematic Instruction

Often, the term *systematic* is heard alongside *explicit instruction*. It is important to specifically define systematic instruction as a means for understanding how it complements explicit teaching strategies. The *2021 IES Practice Guide: Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades* states that "The term systematic indicates that instructional elements intentionally build students' knowledge over time toward an identified learning outcome" (Fuchs et al., 2021, p.12). Recommendations for implementing this practice include integrating previously learned content, sequencing instruction logically so that learning builds incrementally, including visual supports, and providing immediate corrective feedback.

Differentiated Instruction

Differentiated instruction offers an organized way of proactively adjusting teaching, learning to meet kids where they are, and helping them to achieve maximum growth as learners. Differentiation of teacher-directed instruction is a teacher's response to learners' needs. It is guided by general principles of differentiation, such as the use of data, sequence of instruction, flexible grouping, materials and resources, and teachers' and coaches' collaboration in planning. It involves using multiple approaches to the content, process, product, and learning environment. Teachers can differentiate what students are learning, how students are learning, how students demonstrate their learning, and the climate of the classroom (Tomlinson, 1999; Tomlinson, 2014). For more clarity regarding differentiation, see our KS MTSS Math Repository.

Scaffolded Instruction

Scaffolded instruction is "the systematic sequencing of prompted content, materials, tasks, and teacher and peer support to optimize learning" (Dickson, Chard, & Simmons, 1993). When students are learning new or difficult tasks, they are given more assistance. As they begin to demonstrate task mastery, the assistance or support is decreased gradually in order to shift the responsibility for learning from the teacher to the students. Thus, as the students assume more responsibility for their learning, the teacher provides less support. For students to become proficient in performing mathematical processes, explicit instruction should include scaffolded practice, in which the teacher plays an active role and gradually transfers the work to the students. This phase of explicit instruction begins with the teacher and students solving problems together. As this phase of instruction continues, students should gradually complete more steps of the problem with decreasing guidance from the teacher. Students should proceed to independent practice when they can solve the problem with little or no support from the teacher (Gersten et al., 2009a).

Hamilton and Amador (2019) recommend a three-step sequence for scaffolding: prompt-cuereteach. Prompting type questions serve to activate students' prior knowledge. Cues direct students to the necessary information; however, one must be mindful not to jump straight to cueing prior to giving the necessary prompts. Finally, reteaching is a last resort when prompts and cues have proven ineffective.

Computational Fluency

According to the National Mathematics Advisory Panel (2008), the ability to recall basic mathematics facts is critical for general success in mathematics. The National Mathematics Advisory Panel (2008), Common Core State Standards (n.d.), and National Council of Teachers of Mathematics (NCTM, 2006) all state that the quick and accurate recall of math facts is a core skill and prerequisite for higher-level learning. Automatic recall must be developed over time through sufficient instruction, practice, and feedback (Baroody, 1999; Willingham, 2009). VanDerHeyden and Burns (2008) describe this process as moving from acquisition (accuracy) to proficiency (speed). The IES guide (2009) recommends that interventions at all grade levels devote ten minutes in each session to building fluent retrieval of basic arithmetic facts (Gersten et al., 2009a). The Kansas MTSS and Alignment recommends that all students, whether they are receiving intervention or not, undergo this ten-minute differentiated period devoted to arithmetic automaticity. For more information on improving students' computational fluency, refer to the 10 Minutes of Computational Fluency brief.

Mathematics Vocabulary

The number of mathematics vocabulary terms that students encounter throughout grades K-12 merits the intentional focus on this particular topic. The *2021 IES Practice Guide* recommends that educators dedicate adequate time to teaching mathematical language in order to support students in effectively communicating their conceptual understanding (Fuchs et al., 2021). The Standards for Mathematical Practice call for educators to create an instructional environment that expects students to routinely construct viable arguments, critique the reasoning of others, and attend to precision. The intentional alignment of systematic and explicit vocabulary instruction across grade levels and content areas plays a critical role in carrying out those practice standards to address gaps and improve coherence between core and intervention (Nelson et al., 2019a).

Supplemental Instructional Practices

Computer-Assisted Instruction

While computer-assisted instruction (CAI) programs can offer instruction, drill and practice, and motivation, which can all be beneficial for students, CAI is not a replacement for direct, explicit instruction from a qualified classroom teacher (Codding et al., 2017). We recommend that teachers provide explicit instruction for conceptualization and accuracy prior to students using CAI. Most CAI programs address word-problem solving and computational fluency, with fluency being the prominent component. Rich et al. (2017) compared second grade students' abilities to generalize their math facts for different assessment formats, dependent upon the type of practice utilized. Students' practice opportunities consisted of computer-based only, paper/pencil only, **or** a combination of the two. Students who received paper/pencil practice or a combination made significant gains from pre- to post-test on both assessment formats (computer vs. paper/pencil). The students who only received computer-based practice struggled to apply those skills to the paper/pencil assessment. One can conclude that only offering computer-based practice can hinder students' ability to generalize those skills to different formats, which is an important stage in the instructional hierarchy; acquisition, fluency building, and generalization (Haring et al., 1978).

Other considerations for CAI include evaluating the capacity for technology use (e.g., number of computers available, broadband) within the building or district and training of staff to ensure fidelity of implementation, funding, and scheduling (minutes per day, days per week). Teams should look for the aspects below when choosing a CAI program (Codding et al., 2017):

- Evidence-based teaching practices including explicit teaching and scaffolding
- Alignment to core curriculum (strategies, vocabulary, etc.)
- Data-based individualization, tracking mastery
- Immediate, corrective feedback to students
- Engagement and motivation for students

Websites that review and/or list CAI programs:

- LearningWorks for Kids
- TechMatrix
- Common Sense Education
- What Works Clearinghouse
- Educational Technology Clearinghouse, University of South Florida
- The Math Forum, Drexel University

Cooperative Learning Strategies

Visible Learning research assigns an effect size of 0.53 to cooperative learning and recommends this practice for both gifted and remedial learners. Perseverance with mathematics tasks and developing students' productive disposition proficiency strand can both be nurtured through cooperative learning. Zakaria et al. (2013) found that secondary students working in a "cooperative group were able to increase their understanding and to develop their self-confidence" (p. 1). Other benefits to cooperative learning include increasing social skills, self-efficacy, effort, and participation (Codding et al., 2017).

Peer-Assisted Learning

Peer tutoring has the potential to accelerate learning with an effect size of 0.66 according to the Visible Learning research. Students work in pairs and can switch roles back and forth from tutor to tutee. The larger concept of peer-assisted learning includes students working in small groups or teams. According to Codding et al. (2017), important considerations for peer-assisted learning include:

- Close monitoring by the teacher in order to provide immediate feedback
- Training of students in the necessary skills and roles by modeling, role-playing, and

guided practice with the teacher
- Setting group goals and providing rewards for collaboration, effort, and academic performance
- Establishing student expectations during peer tutoring sessions

Math PALS is an example of an evidence-based, whole-group, peer-tutoring program for grades K-6.

Maintenance Practice

A common theme or concern heard amongst mathematics educators is that students are not maintaining previously taught skills. Since mathematics concepts build upon each other, it is important for students to retain critical skills from lesson to lesson, unit to unit, and grade to grade. Watch the video Maintenance Practice in Mathematics Classrooms with Dr. Paul Riccomini for training on this subject. The following practices are effective ways to increase retention:

Retrieval Practice

Retrieval practice involves giving students opportunities to recall, from memory, previously learned information. This recall opportunity is then followed by feedback so that students can rate themselves on how well they recalled the information. Lastly, it is important for this practice to be low stakes; "Think of retrieval as a learning strategy, not an assessment tool" (Agarwal et al., 2020, p.3). See Retrieval Practice Guide.

Interleaved Practice

Mathematics textbooks typically use what is called *blocked practice*, which occurs when students use the same strategy or procedure for multiple consecutive problems (Rohrer et al., 2017). Dr. Riccomini shares that this practice is appropriate when students are acquiring a new skill or concept; however, for maintenance of previously learned skills, it is more effective to incorporate *interleaved practice* opportunities in addition to blocked practice (Ragsdale, 2020). Interleaved practice is simply mixing problem types so that consecutive problems require students to choose a different strategy or procedure. See Interleaved Mathematics Practice Guide.

Spaced Practice

If you have ever crammed at the last minute for an exam, then you understand the idea of massed practice. You might have noticed that, shortly after the test, you were unable to recall the majority of the information. This is because "when information is quickly acquired, it's often quickly forgotten" (Carpenter & Agarwal, 2020, p.5). *Spaced practice* involves chunking a lesson or practice session over time into multiple, shorter sessions. Carpenter and Agarwal state that this method allows students to transfer the information to long-term memory and therefore increase their ability to retrieve that information in the future. See Spaced Retrieval Practice Guide.

Metacognitive Strategies

Visible Learning defines meta-cognition as *thinking about thinking and includes methods used to help students understand the way they learn*. Meta-cognition has been assigned an effect size of 0.52, meaning that it has the potential to accelerate learning. When students reflect on what they know and monitor their own learning, while also using those reflections to inform their next steps, they are practicing the key components of meta-cognition (Son, Brittingham-Furlonge & Agarwal., 2020). Retrieval practice and spaced practice are two ways to improve students' meta-cognition and therefore improve their learning. See Metacognition Guide.

Classwide Intervention

Generally speaking, when more than 40% of students in a particular class do not meet benchmark on a universal screening measure, then it is recommended that educators implement a classwide intervention. Reminder: a strong tier 1 is the first and best intervention. Skills targeted during a classwide intervention should reflect skills that students have acquired but in which they need to build fluency. The National Association of School Psychologists recommend the following active ingredients to building a successful classwide intervention (2020, p.2):

- Guided practice with corrective feedback as needed.
- Think aloud during problem solving.
- High dosage of opportunities to respond at the correct level of task difficulty.
- Task difficulty is selected to reflect a skill that the student has acquired (i.e., the child can accurately complete the task, but the performance is labored).
- Independent practice with a goal to try to "beat one's last best score."
- Delayed error correction and explanation to the math partner how the error was corrected.
- Group contingency delivering a small privilege, reward, or celebration based on the growth of the class as a whole.

Spring Math is an example of a classwide math intervention program. MIND: Facts on Fire is a free, schoolwide (Tier 1) application for addressing acquisition and fluency building of basic math facts.

Other evidence-based strategies for implementing a classwide math intervention for computation and/or word-problem-solving deficits include the following:

- Cover-Copy-Compare
- Taped Problems
- Schema-Based Instruction

Tier 2 and 3 Mathematics

Scheduling Considerations, K-12

- Avoid conflicting with core, other content, and recreational periods. **Intervention time should stand alone** and be represented as a separate slot of time within the master schedule.
- Ensure 50-60 minutes of core instruction, of which **10 minutes is devoted to differentiated computational fluency practice**.
- Ensure that intervention time is sufficient to accommodate the recommendations of the chosen curriculum. If you have not chosen a curriculum, plan for approximately **30 minutes at least four days per week**.
- If math time must be split, consider splitting the core time into more than one section of time prior to chunking intervention time.

Resources and Recommendations for Math Intervention

One of the challenges facing the leadership team is to identify resources that might already be available in the system to provide effective interventions for students. IES also provides multiple practice guides with evidence-based recommendations from panels of national experts. These recommendations not only support students who are struggling in mathematics but are also strong practices for supporting the mathematics proficiency of all students. The Kansas MTSS and Alignment Math team encourages educators to consider embedding the instructional practices from the following IES practice guides:

- *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades*
- *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*
- *Improving Mathematical Problem Solving in Grades 4 through 8*
- *Teaching Strategies for Improving Algebra Knowledge in Middle and High School Students*
- *Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools*

Assessment

Assessment plays a key role in determining the effectiveness of instruction. “Instruction is successful, or effective, to the degree that it accomplishes what it sets out to accomplish” (Kirschner, 2022). Assessment measures the success of instruction and therefore the overall purpose of mathematics assessment must be to improve student learning. Assessment should support the learning of important mathematics and furnish useful information to both teachers and students. NCTM (2007) maintains that assessment should be an integral part of instruction, providing not only the teacher but also the student with information about the student’s learning.

Universal Screening

Universal screening assessments must be reliable, valid, and efficient. Specific recommendations for criteria for these features can be found in the *IES Practice Guide: Assisting Students Struggling* *with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools* (Gersten et al., 2009a). It is recommended that a universal screening assessment take less than 20 minutes to administer. Universal screening in MTSS addresses basic critical skills/concepts, and not every concept is taught in the classroom. Remember that universal screeners are formative, and their purpose is to identify students at risk of not meeting current and future benchmarks.

Universal Screening for Grades K-1

Universal screening for kindergarten and first grade assesses the skills and concepts related to number sense. Measures typically include constructs of numeral recognition (number identification), magnitude comparison (quantity discrimination), and strategic counting (missing numbers). In some assessments, strategic counting and magnitude comparison have been identified as key predictive variables (Gersten, Clarke, & Jordan, 2007). All students in kindergarten and first grade should also be screened for early numeracy skills three times a year.

The early numeracy assessments are often administered individually and typically take one minute per subtest. Students who fail to reach the benchmark on one or more of the early numeracy subtests are grouped for instruction during the MTSS implementation process and sorted into groups for intervention for early numeracy skills.

Universal Screening for Grades 2-12

All students in grades 2-12 should be screened three times per year. Universal screening measures for math can be given to an entire classroom and do not require individual administration. The screening data will identify students who are at, above, or below benchmark. Students who are below benchmark will be further grouped for intervention.

Diagnostics

Informal Diagnostics

The purpose of an informal diagnostic is to identify the instructional focus for each student requiring intervention support. Tools that can be used to identify the instructional focus for students include placement tests within the intervention curriculum, reports within the screening assessment system, pre/post measures within the core curriculum, student interviews, and/or student work samples.

Formal Diagnostics

Formal diagnostics are designed to be used for students who are not progressing as intended and need to be assessed for specific misconceptions that might not have been evident in previous measures. Formal diagnostic assessments for mathematics provide a more in-depth analysis of a student's strengths and weaknesses and are used to further guide instruction. Most diagnostic assessments will provide either age-based or grade-based norms or rubric scoring, which are used to determine whether a student has significant problems in specific skill domains. This information can then be used to design instruction specific to the student's individual learning needs. It is important that formal diagnostic assessments be given to students when additional information is needed for more customized instructional planning, but it is also important not to overuse these assessments. Formal diagnostic assessments require numerous building resources and should not be given as a matter of course to all students. Instead, they should only be given when their progress monitoring data indicates that further information is necessary to adequately plan instruction.

Single-Skill CBM Probes and Error Analysis

Whenever students fail to make adequate growth in intervention, it might be an indication that further analysis is needed. The school might decide to gather additional diagnostic information by conducting error analysis and examining error patterns (Ashlock, 2006; Riccomini, 2005), especially at the student's instructional level. When error analysis indicates possible skill deficits, verification of these deficits can be conducted by using single-skill CBM probes (Hosp, Hosp, and Howell, 2007). Each single-skill probe assesses only one type of skill at a time, enabling a more reliable and valid assessment of specific deficits for a given computational skill. Single-skill CBM probes for mathematics are available from Intervention Central. Marilyn Burns' Listening to Learn math interviews provide a source for error analysis.

Comprehensive Formal Diagnostic Assessments

The school might also choose to administer a formal diagnostic assessment to determine underlying mathematics issues. This assessment is not necessarily for special education referral, but rather for the purposes of planning instruction. Understand that these types of assessments are dense in nature and will take training and larger amounts of instructional time to complete*. It is important that these assessments only be used with students who truly need them*. Less-invasive diagnostic information includes but is not limited to suggestions listed previously as well as an integrated look across reading, behavior, and social needs. This information should be considered prior to administering a formal diagnostic. The team should also ensure that initial grouping was appropriate.

Formal diagnostic assessments for mathematics include but are not limited to:

- TEMA-3 – Test of Early Mathematics Ability–Third Edition (Between Ages 3.0 and 8.11)
- Key Math III (grades K-12)
- Tools for Early Assessment in Math (TEAM) (grades preK-2)
- Number Knowledge Test (age levels 4, 6, 8 and 10 years)

Decision Rules for Formal Diagnostic Assessments K-12

All buildings should establish decision rules to address when additional diagnostic assessments will be given, which students will receive tiered support, and how students will be assigned to skill groups. There could be different decision rules established for the use of brief, criterion-referenced diagnostic assessments as compared to more formal, norm-referenced diagnostic assessments that are more resource-intensive to administer. The leadership team should then review each selected diagnostic assessment to determine the skills/concepts assessed and time to administer.

Progress Monitoring

Formal progress monitoring refers to a broad assessment or General Outcomes Measure (GOM), ideally the same broad construct as the universal screener, which will identify how the student's general math ability is progressing in a normed comparison across other students at the same instructional level. *Informal progress monitoring* refers to mastery assessments within the intervention curriculum in the form of pre-/post-testing. Both are critical and vary in interpretation and application. We recommend administering a *formal progress monitoring measure* every two to four weeks and skill-specific *informal progress monitoring measures* as often as needed in order to inform instruction and gauge students' mastery.

Math Grouping Process

According to universal screening results, students who score at or above benchmark are considered Tier 1 and perform at their current grade level. Students who score below benchmark are considered Tier 2 or 3. ALL students, *including* those in Special Education, receive 50-60 minutes of differentiated core instruction every day. Students needing Tier 2 or 3 support will also receive a minimum of 20-30 minutes of intervention at least four days per week. If utilizing a walk-to intervention model, students at Tier 1 should receive enrichment opportunities during this time.

Identifying Instructional Level for Tier 2

Students receiving Tier 2 support are to be considered below benchmark, but are most likely still on level with their current grade (i.e., a student who tests at a Tier 2 level in third grade will have an instructional level of grade 3). However, it is important to note that secondary students receiving Tier 2 supports could have deficits below their current grade level.

Identifying Instructional Level for Tier 3

Backwards Testing

Students at Tier 3, however, must undergo a series of backwards tests to determine their instructional levels. In some assessment systems, this will require the student to take the previous grade's universal screener to compare achievement with that level's end-of-year benchmark score. If the student does not achieve the end-of-year benchmark score, he/she will then take the previous grade universal screener. This process will continue until the student achieves at or above the end-of-year benchmark score. Once this occurs, a student will be considered to have an instructional level at one grade above that achievement. For example, if a fifth grade student tests backwards and they achieve above the end-of-year benchmark on the second-grade screener, their instructional level is third grade.

Scaled Scoring

Other systems use an adaptive screening assessment that will provide a student a scaled score that will be consistent through all levels (K-8). Finding the student's scaled score at the assessment's recommended percentile at subsequent grades will enable educators to find their instructional level. For example, an eighth grade student scores a 212 on the FastBridge aMath screener, and the 50th percentile scores for fall of fourth and fifth grades are 210 and 216, respectively. Therefore, this student's instructional level is fourth grade.

Identifying Instructional Focus for Intervention (Tiers 2 and 3)

Once instructional levels are determined, a comprehensive protocol placement test within the selected intervention curriculum should be utilized to identify the specific point of beginning intervention (instructional focus). If the intervention curriculum does not contain a placement exam, it might have an instructional planning report that can be used to determine a starting point. Leaders might also consider using other diagnostic measures (See *Informal Diagnostics* above). Students are then placed into homogenous intervention groups based on their identified instructional focus. These groups are not static, but should be fluid in nature to allow students to progress and move through the continuum of critical math skills. This process is highly complex and involves student-level decision making due to the overlapping skill deficits along the progression of the mathematics standards. It might be necessary to utilize more than one diagnostic tool to narrow the instructional focus for each student.

Professional Development and Fidelity

Professional Development Considerations

The Building Leadership Team will identify the professional development needs of staff related to all mathematics curriculum, instruction, and assessment. *All* staff members with instructional responsibility must have a solid understanding of the *core curriculum* and receive professional development that enables them to implement it with fidelity. In this instance, staff refers to the staff members responsible for instruction at all three MTSS levels. This is necessary to ensure that the intervention curriculum is aligned with the core curriculum and all students have equal opportunities and access to high quality instruction independent of their classroom placement.

It is not necessary that all staff members in a building know how to implement intervention curricula; however, everyone involved in collaborative teams should understand the skills targeted in each curriculum so they can be involved in instructional planning.

The most effective intervention teachers are those who are trained in the use of a well-developed, explicit, and systematic math intervention program and are provided ongoing support for fidelity of implementation. Students with individualized education plans are **not** excluded from access to small-group instruction. Teachers of these groups can include general education teachers, paraprofessionals, specials teachers, non-math content teachers, Title teachers, and special education teachers. Instruction can occur both inside and outside the special education classroom and should be based on a common identified instructional focus. Remember: special education is a service, not a room. The KSDE Special Education and Title Services asserts, “those children who do not respond to the core instructional procedures will receive targeted group interventions in addition to core instruction” (KSDE, 2019, p. 26).

Often when determining the capacity of school staff utilized during math intervention, we rule out non-math content teachers, thinking that the lack of mathematics knowledge will impede student achievement. Recent research, however, deems otherwise. Nelson, Van Norman, Parker, and Cormier (2019b) found no effects on students in grades four through six; math achievement scores were observed for interventionists’ content knowledge. A stronger factor in student achievement, rather, was the fidelity of implementation of the mathematics intervention. “Higher student post-test scores were observed for interventionists with an average fidelity of 95% or greater" (Nelson, Van Norman, Parker, & Cormier, 2019). For this reason, it is recommended that districts have a system of training and ongoing evaluation in place for measuring implementation fidelity of math interventions. In practicality, these findings broaden the capacity for staffing intervention, rather than districts being limited to only math content teachers.

When determining interventionists, it is critical to have a good match between the instructors and the interventions they will be teaching. Leadership teams should consider how all staff members can be utilized. For example, if a particular teacher is highly skilled in understanding the skill progression underlying operations with fractions, allow them to utilize that strength for the benefit of the students in that particular skill-based group.

If your district has questions or needs guidance regarding implementation fidelity, refer to the Kansas MTSS & Alignment Phase 1 Guide or reach out to your regional Systems Alignment Specialist.

Ensuring Fidelity of Assessment

Professional development for the selected assessments should be integrated into the district and/or building's professional development plan and go beyond the assessment training to include ongoing support, coaching, and onboarding to ensure sustained fidelity. Decisions need to be made about who will administer, score, and interpret each assessment. All staff members involved in the administration of an assessment need to be trained on the purpose, rationale, and uses of the assessment and how to interpret the data and its instructional implications. Not only does this help build school capacity, but it also encourages buy-in of the assessment, which is critical for ensuring that teachers use the data to inform their instruction.

Staff members should share a common understanding of monitoring fidelity, viewing it as a means of leveraging their collective impact rather than a punitive evaluation process. The monitoring of fidelity ensures that all data is appropriately collected and used. The three main factors that need to be monitored are as follows: all staff members are trained to administer and score assessments, decision rules and assessment calendars are followed, and results are correctly interpreted and used to guide instructional planning.

Training for staff members is best scheduled just before the assessments are given so the scoring rules can be practiced and reinforced, depending on the assessment system. Effective ways to minimize scoring errors and ensure fidelity include making sure that examiners have excellent training and practice opportunities, periodic ongoing training, experienced examiners to check first-time examiners' scores, and opportunities to shadow score.

Ensuring Fidelity of Curriculum and Instruction

The professional development plan for curriculum implementation proactively identifies training based on staff learning needs. This ensures that staff members access and utilize curricular materials in the expected manner. To accomplish this, leadership teams should establish methods for monitoring the use of the curriculum by individual teachers. This monitoring allows leadership to customize and plan ongoing professional learning opportunities to support each staff member.

Activities for monitoring the individuals' fidelity of curriculum implementation are not intended to be punitive, but rather should be understood as a piece of the overall professional development plan, resulting in further staff support as needed. To accomplish this, a method to check for the correct use of the curriculum materials must be established. Many purchased curricula and programs come with fidelity-monitoring tools such as observation or walk-through forms. If districts desire, they can create a form or checklist of their own.

Depending upon district preference, district or building leadership teams might be responsible for establishing a plan to monitor and support the correct and effective use of curriculum materials and instructional practices. The following steps can be used to decide how to support staff in the use of evidence-based materials and instructional practices:

- Develop a plan to provide professional development to appropriate instructional staff (e.g., ESOL, Migrant, Title, SPED, and paraprofessionals)
- Determine the key elements of instruction that need to be monitored for fidelity and who will be monitored
- Determine a method (e.g., walk-through, peer coaching) to monitor key elements for fidelity as well as the frequency
- Develop and implement a plan to provide training and coaching to instructional staff members who need additional assistance in providing instruction, as identified through monitoring
- Monitor the plan for fidelity of implementation

These critical components are designed to help leadership teams as they begin the development of an overall professional development plan. Once specific decisions are made, the building leadership team should record the results on the building's professional development plan. The leadership team should also consider whether the discussion of professional development and fidelity of instruction has led to a need to develop an action plan.

For more information regarding how to plan for professional development and monitor fidelity, refer to the Phase 1 Guide

Leadership

Building Leadership Team

the building leadership team (BLT) is an essential component of the district's self-correcting feedback loop, because this team ensures communication between the collaborative teams and the district leadership team. Then the building leadership team also ensures fidelity of implementation, effectiveness of the system, and monitoring of student progress. The team should include teachers, administrators, and staff members who are empowered to make decisions and have areas of expertise that contribute to the academic and social emotional growth of the students.

Collaborative Teams

The purpose of collaborative teams is to review data related to student improvement, share data and ideas, and collaborate with other teams to refine instructional methods. Depending on the district, each collaborative team can include but not be limited to every teacher of the same grade level or band, every teacher of the same content area, special education teachers, classified staff, specialists, and coaches. It is also recommended that each collaborative team include a representative from the BLT in order to ensure effective communication within the self-correcting feedback loop.

Empowering Culture

Belief in Students

The first essential component in mathematics is that teachers hold an ingrained belief that ALL children *can* achieve proficiency with mathematics. Wilkerson (2020) underscores the importance of this belief in an NCTM President's Message, titled *Believing Our Students Can Do Mathematics*, by posing the following questions:

- Do I really believe that each and every student can do mathematics?
- Do I believe each and every student can engage in rigorous mathematics and problem solving?
- Do I believe that each and every student can contribute to classroom discourse about rich mathematical concepts?

Collaborative teams are highly encouraged to truly reflect on their beliefs and how they impact their students. Teachers must support each other and hold each other accountable in upholding these beliefs. However, this approach goes beyond beliefs because they should translate into actions. Do teachers' beliefs and actions align? With the support of building leadership, collaborative teams must have these crucial conversations.

Collective Teacher Efficacy

Visible learning research defines Collective Teacher Efficacy (CTE) as "the shared belief by a group of teachers in a particular educational environment that they have the skills to positively impact student outcomes." CTE has the potential to more than triple the rate of learning for students; however, it is neither a quick fix nor easily achievable. Nurturing CTE in a school building and district takes time, but it is well worth the investment. Donohoo (2017) describes four sources that shape this shared belief by teachers as mastery experiences, vicarious experiences, social persuasion, and affective states. The webinar Fostering Collective Teacher Efficacy expounds on this topic. Mastery experiences prove to hold the most value, and this is why the Kansas MTSS and Alignment framework advocates for the incorporation of short impact cycles when implementing evidence-based practices as one way to build CTE. See the Phase 1 Guide for more details about impact cycles.

Family Engagement

Bryk et al. (2010) found that schools that are chronically weak in family engagement did not improve math scores. In contrast, schools strong in family engagement were ten times more likely to improve math scores. We recommend partnering with parents to build a deeper understanding of the mathematical concepts and procedures their students will be learning each year. By intentionally sharing student data with parents, educating them on student assessments, and collaborating to set students' goals, educators take the lead in improving communication between school and home. To learn more about ways to involve families, please visit the Kansas Parent Information Resource Center.

Implementing the Kansas MTSS & Alignment Framework

Kansas MTSS & Alignment Implementation Steps

Step 1: Review and Validate Universal Screening Data

Step 2: Analyze Data

Step 3: Use Data to Group Students

Step 4: Determine Focus of Intervention

Step 5: Progress Monitoring

Step 6: Document Interventions

Defining Each Implementation Step

Step 1A: Review and Validate Universal Screening Data, Validity at the System Level

In addition to considering the validity of scores for individual students, the building leadership team should review systemic issues that could affect the validity of screening data. The building leadership team must also review the fidelity of administration of the universal screening assessment by discussing and reviewing any information collected regarding the following issues:

- Were the directions for the administration of the screening assessment followed exactly?
- Were the time limits for each test followed exactly?
- Was shadow scoring used to check scoring fidelity?
- Was the assessment calendar followed?
- Have all staff members who administer the assessment been trained?

Step 1B: Review and Validate Universal Screening Data, Validity at the Student Level

The following are some questions that the collaborative teams should consider when validating the screening results:

- Was the screening assessment administered with fidelity?
- Were there environmental circumstances or events in the student's life that could have impacted score results? For example, was the student sick on the day of the universal screening assessment? Had a traumatic event occurred recently?
- What other reasons can be identified for a lack of confidence in the score? For example, does the student exhibit inconsistent patterns of performance across data collection events? In other words, are there student characteristics that we need to consider when interpreting the results of a specific assessment?

Step 2: Analyze Data

The purpose of analyzing data as a building leadership team is to have building-wide, system-level discussions, by looking at what universal screening data is currently available. After every universal screening administration, the building leadership team will review building-level data to determine if the core curriculum has sufficiently met the needs of most students (80% or more students at or above benchmark) and, if not, to provide a general understanding of how many students might need additional Tier 2 or Tier 3 support from the system. There should also be an intentional effort to communicate the needed PD or other issues to the district leadership team, as there could be district-wide issues that need to be addressed, or the district might need to allocate resources differently.

If fewer than 80% of students meet the benchmark, several potential causes should be considered:

- Are core instruction and core curriculum being implemented with fidelity? How do you know?
- Is core instruction taught using evidence-based practices?
- Are concepts being taught to mastery?
- Are there sufficient examples, explanations, and opportunities for practice to support new learning?
- In terms of differentiating the core, what thoughts arise with regard to the strengths and needs of district staff?
- Are professional development or supports needed for teachers regarding the core curriculum or instruction?

When analyzing mid-year screening data, it is important for teachers to look at individual student growth. Often a student who is performing well below grade level will score at risk on the mid-year screener; however, if the student has made typical or aggressive growth (>40^{th} percentile growth), then the collaborative team should consider maintaining the current intervention plan. Regardless of a student's risk level at mid-year, students should be making typical or aggressive growth from fall to winter. If a student is making moderate or flat growth (<40^{th} percentile growth) or digressing, then the collaborative team needs to problem solve possible reasons behind the lack of growth. Student attendance, teacher attendance, intervention focus and/or strategies, intervention dosage, and group size are some factors to consider when a student is not making adequate growth. In these cases, it is the collaborative team's responsibility to revise or intensify the current plan, as needed, in order to improve students' trajectory toward the end-of-year benchmark.

Steps 3 & 4: Use Data to Group Students and Determine Focus of Intervention

Once the universal screening data has been analyzed and validated, collaborative teams should work together to gather additional needed information, as outlined in this section, and group students appropriately for intervention. All students, kindergarten through 12^{th} grade, take the universal screener three times per year. This includes students receiving special education services, title services, and ELL support. Below is a general process for using data to group students, determine focus of intervention, and progress monitor. (Access math grouping flowcharts and descriptions for specific assessment systems here.)

Tier 1

Students who fall under the Tier 1 category are at or above their benchmark according to their universal screening score (e.g., aMath, MAP Growth, earlyMath, Concepts & Applications), meaning they scored at or above the 40^{th} percentile. *All* students should receive 50 to 60 minutes of core instruction with differentiation.

Tier 2

Students who fall under the Tier 2 category are below the benchmark according to their universal screening score. These students should receive 20-30 minutes of intervention at least four days per week beyond their daily 50 to 60 minutes of core instruction. Use a placement test to determine the instructional focus for each student's intervention time. If no curriculum placement test exists, use other diagnostic information including any relevant reports within the assessment system. Group students based on instructional focus and begin comprehensive protocol intervention. Progress monitor on grade level every two to four weeks. If students are making adequate progress, continue the intervention. If students are not making adequate progress, conduct an error analysis and/or formal diagnostic in order to customize and continue the intervention.

Tier 3

Students who fall under the Tier 3 category are well below benchmark. These students should receive 20-30 minutes of intervention at least four days per week, in addition to their daily 50 to 60 minutes of core instruction. Use norms charts within the assessment system to determine each student's instructional level. Next, use a placement test at the determined instructional level to identify the instructional focus for each student's intervention time. If no curriculum placement test exists, use other diagnostic information including any relevant reports within the assessment system. Place students in groups based on instructional focus and begin comprehensive protocol intervention within critical areas of mathematics that ensure future success in algebra. As teams begin to document students' placement, it is imperative that students be recorded in such a way to ensure that those with similar mathematical deficits are grouped together.

Developing a process for grouping students and determining instructional focus for math are somewhat more complex tasks than those applied for reading. It is important to note that fluency/accuracy grouping cannot be used for math in the same way that it is for reading. In addition, because the math proficiencies are completely intertwined, a comprehensive approach to intervention is often more advantageous than addressing a single skill/concept.

Fluid Grouping

While it might not be necessary to restart the grouping process at each benchmark period, whenever a universal screening is conducted, it is essential to revisit and refine the alignment of intervention groups. Analysis of the current data and progress monitoring groups in light of the newly established benchmark data is critical to ensure that the current groups contain homogeneous instructional levels and foci.

Further Instructional Considerations for Intervention

Fluency

Computational fluency appears to be an underlying issue for many students, and the *2009 IES Practice Guide: Assisting Students Struggling with Mathematics: Response to Intervention (RtI) for Elementary and Middle Schools* recommends that "Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts" (Gersten et al., 2009). The Kansas MTSS and Alignment, for good reason, has expanded this 10-minute devotion to all students in grades K-8 (read Computational Fluency Brief), but it could also be appropriate for high school students that exhibit deficits with basic computation. Fluency instruction and practice should be differentiated for each student, based on where he/she is currently performing with basic facts. Differentiation cannot be stressed enough within this practice time. Content and instruction must be individually tailored to best ensure promising practice. Especially when working with students to build proficiency or automaticity, teachers should consider how information might be chunked or grouped into smaller pieces for instruction (Riccomini & Witzel, 2010). Strategic progressions and chunking of fact families and/or like strategies (e.g., doubles, near doubles) can help avoid overloading students' processing capacity (working memory) while facilitating conceptual understanding of specific fact combinations rather than solely memorization. According to the instructional hierarchy, acquisition (accuracy) precedes fluency building; therefore, individual practice (timed worksheets, computer-based programs, etc.) should only be utilized once students have shown accuracy with little adult support (Haring et al., 1978).

Timed Activities

Data emerging from the 2022 National Assessment of Education Progress (NAEP) shows that only 33% of fourth-grade students nationally performed at or above the NAEP proficient level on the mathematics assessment, while 75% percent of fourth-grade students performed at the (lower) basic level. Performance levels relate to mastery of specific standards that describe what a child should know and be able to do at a specific grade level (NAEP, 2023). By grade eight, the percentage of students deemed proficient drops to 26%, and by grade twelve, 24%.

At the very foundation of mathematics are hundreds of declarative addition, subtraction, multiplication and division facts. Providing instructional conditions that are ripe for students mastering these basic facts is critical and must be placed as a priority within a school system. But what does it mean to have *mastered* these facts? "Fluency refers to the speed with which students can accurately perform a skill. Because fluency is composed of both speed and accuracy, a student who performs a skill quickly but inaccurately is not fluent, and neither is a student who performs a skill accurately but slowly" (Datchuk & Hier, 2019). Dan Willingham (2009, p. 16-17) writes that, for the basic facts "of addition, subtraction, multiplication and division, answers must be well learned so that… the answer is not calculated but simply retrieved from memory… [A]utomatic retrieval of basic math facts is critical to solving complex problems…. Calculating simple arithmetic facts does indeed require working memory." Hartman, Hart, Nelson, and Kirschner (2023, p. 11) argue more firmly that, "Substantial class time must be budgeted to not just learn but *overlearn* these over 240 verbatim fundamentals." What constitutes whether or not a fact is automatic (not calculated)? Does evidence point to a specific instructional method that has been shown empirically to be effective?

When a fact is *automatic*, it is firmly committed to memory. When a student is *automatic*, no outside strategies are used or fingers counted, but they may be used when a student is gaining mastery of accuracy within the acquisition phase of learning (Haring et al., 1978). Instead, the student sees two numbers and instantly knows the sum, difference, product, or quotient. Imagine the impact such an outcome would have on a student's ability to accurately perform algorithm-based mathematical problem-solving both now and into the future!

In order to determine how to increase fluency and automaticity in basic fact retrieval, teachers should look to empirical research to find instructional methods that have demonstrated effectiveness across thousands of students and multiple contexts. Newly released in 2021, the Institute of Education Science appointed a strong level of evidence to the effectiveness of timed activities for supporting automatic retrieval of basic facts (p. 51) and also provided recommendations for implementing the practice:

1. Identify topics a student has already learned (the student is accurate)
2. Select the activity, procure materials, and set clear expectations. Timed activities are intended to be brief but require a student to produce multiple correct answers in a short amount of time. Activities in which fluency building may include skill-based worksheets, computer programs, or flashcards, and they can be structured for both individual, partner, or group work. Since the students have already become accurate with the targeted facts, frustration should be low.
3. Ensure students have a strategy that is efficient prior to completing the timed activity.
4. Motivate and encourage students to work hard by having them chart their individual progress.
5. Provide immediate feedback and ask students to correct errors using a strategy that is efficient.

"Giving timed worksheets alone does not support fluency" (IES, 2021, p. 55). The goal is not to hand out fluency worksheets every day; that alone will not bolster automaticity. It is vital to consider where a student is in their learning as it relates to the Instructional Hierarchy (Haring et al., 1978) and to match the activity to that specific learning phase. For example, if a student is working to become accurate with a small set of basic facts, the activity matched with that specific phase of learning is not one aimed at gaining speed; rather, the activity would have a goal of improving accuracy (e.g., slowing down to become accurate). If students are becoming frustrated or anxious over the timed activity, chances are that either the set of problems needs to be reduced, or further analysis should take place to identify where the student is in their learning of that particular set of facts, perhaps stepping back to ensure accuracy first.

Considering the number of practice opportunities students need for basic facts to become deeply rooted, or overlearned, within one's memory, it is vital to consistently interject these practice opportunities within daily math instruction. Building automaticity takes time, but the reward is long-lasting.

Fractions

For students at the intermediate and secondary levels, additional skill assessment with fractions should be considered for those who score low on any of the screening measures (Riccomini & Witzel, 2010). Under such circumstances, collaborative teams might need to consider more diagnostic information around rational number acquisition, computation, and application in order to determine instruction for an appropriate skill. The *2010 IES Practice Guide: Developing*

Effective Fractions Instruction for Kindergarten Through 8th Grade lists five recommendations to help educators improve students' understanding of fractions. The following examples show moderate evidence:

- Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.
- Help students understand why procedures for computations with fractions make sense.

For further guidance on structuring your math intervention protocol and selecting evidence-based practices, refer to the *2021 IES Practice Guide: Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades*. The IES guide includes six recommendations with strong evidence:

1. Provide systematic instruction during intervention to develop student understanding of mathematical ideas.
2. Teach clear and concise mathematical language and support students' use of the language to help students effectively communicate their understanding of mathematical concepts.
3. Use a well-chosen set of concrete and semi-concrete representations to support students' learning of mathematical concepts and procedures.
4. Use the number line to facilitate the learning of mathematical concepts and procedures, build understanding of grade-level material, and prepare students for advanced mathematics.
5. Provide deliberate instruction on word problems to deepen students' mathematical understanding and support their capacity to apply mathematical ideas.
6. Regularly include timed activities as one way to build students' fluency in mathematics.

Step 5: Progress Monitoring

Monitoring Progress & Monitoring Instructional Levels

Universal screening is always administered at the student's current grade level. Progress monitoring, however, always takes place at the student's instructional level. Progress monitoring students at their instructional level is critical to helping students close the achievement gap between themselves and their peers. The instructional level was determined during the grouping process. That same level should be used for progress monitoring.

Formal vs. Informal Progress Monitoring Measures

The progress monitoring assessments within the universal screening system are considered formal, and it is recommended that educators use general outcome measures (GOM) as they measure growth over an extended period of time. More informal progress monitoring measures, such as skill-based pre- and post-tests, exit slips, checks for understanding, and student work samples, can be utilized to gauge students' mastery of the specific skills they are working on during intervention. Both formal and informal measures are critical for determining the effectiveness of the intervention. Gains on informal measures should produce gains on the broader GOM, or formal measure, over time.

Frequency

The recommended frequency of progress monitoring for math within the Kansas MTSS and Alignment framework is every two to four weeks for students receiving Tier 2 or Tier 3 interventions. Seek to conduct progress monitoring on the instructional level. Consider individual schools' capacity when determining the frequency of progress monitoring, as the data could indicate changes in grouping. Moreover, due to the sensitivity of some early numeracy measures, schools might choose to monitor more often; however, schools should, at a minimum, monitor once per month.

Goal Setting

Begin by setting a goal for the student to achieve the end-of-year benchmark corresponding to his/her instructional level. However, for students who are receiving high-quality intervention, it is appropriate to expect more than a year's growth in a year's time, even if the student has not achieved that rate of growth in the past. Once a student consistently scores above the aim line (considering the most recent consecutive data points), the student should be moved to the next instructional level and the goal adjusted accordingly. Students who score in the Tier 3 range need to set ambitious goals. Research indicates that ambitious goals produce better results than lower goals (McCook, 2006; Hattie & Donoghue, 2016; Sides & Cuevas, 2020). Without an ambitious goal, students in intervention can make progress but continue to lag behind grade level without closing the achievement gap between themselves and their peers.

Analyzing Progress Monitoring Data

Consider these two questions when looking at progress monitoring graphs:

- Is the student growing?
- Is the growth aggressive enough to close the achievement gap?

If students are making adequate progress that will result in meeting their goal, continue the intervention. When students are not making adequate progress, conduct an error analysis and/or formal diagnostic in order to customize and continue the intervention. Prior to conducting a formal diagnostic and/or error analysis, the following questions should be considered:

- Were the appropriate skills/concepts progress monitored at the correct level?
- Has sufficient data been collected to make decisions?
- Was the data accurately graphed?

Exiting Intervention

Students can be exited from intervention once they have consistently met or exceeded the benchmark according to progress monitoring and/or screening data. It is still recommended that instructors administer a formal progress monitoring measure monthly to ensure that these students remain on track to meet the end-of-year benchmark with their peers. Reminder: the benchmark score will continue to increase throughout the year.

Customizing a Math Intervention

When a student receiving intervention fails to show progress despite data-based adjustments, such as increasing dosage, reducing group size, implementing motivational strategies, and increasing opportunities to respond, teams should consider the need for individual student problem solving to customize the intervention. This is the time for teams to decide first to intensify the instruction or to utilize a formal diagnostic assessment to better identify the unknown skill deficits.

Table 1 can be beneficial to this team conversation. While some factors that influence student learning are indeed outside of our control, this chart is meant to identify how teachers can creatively intensify the intervention. For example, if the team believes the student's lack of commitment to school is impacting their math growth, it then becomes the team's job to find a solution to intensify the student's intervention in a way that addresses that issue. Data should then guide whether their decision is effective or if further problem solving must occur.

Table 1. *Research-Based Practices to Consider Regarding Intervention Effectiveness*

<table>
<tr><th colspan="3">Research-Based Practices to Consider Regarding Intervention Effectiveness</th></tr>
<tr><th>INSTRUCTION</th><th>CURRICULUM</th><th>PRINCIPLES OF INTENSIVE INTERVENTION</th></tr>
<tr><td>Fidelity of instruction
Modeling and guided practice prior to independent practice (I Do, We Do, You Do)
Explicit teaching
Opportunities to respond
Sufficient questioning
Check for understanding
Sufficient practice</td><td>Appropriate match between learner and intervention
Appropriate rate of progress to reach goal
Instructional focus based on diagnostic process
Variety of interests
Teaches skills to mastery
Appropriate independent work activities</td><td rowspan="3">Break problems down into smaller steps
Use precise language
Repeat language
Elicit student explanations
Provide explicit modeling
Utilize concrete, representational, and abstract manipulatives
Use worked examples
Provide repeated practice
Engage in error correction
Fade support
Incorporate fluency
Move on
Source: Powell & Stecker (2014)</td></tr>
<tr><th>SETTING</th><th>INDIVIDUAL</th></tr>
<tr><td>Classroom routines/behavior management supports learning
Appropriate person teaching the intervention group
Transitions are short and brief
Academic learning time is high</td><td>Motivation
Task persistence
Attendance
Pattern of performance errors reflect skill deficits
Commitment to school</td></tr>
</table>

If intensifying the intervention does not produce results, a team might determine the need to utilize a formal diagnostic, such as KeyMath3 or Tools for Early Assessment in Math (TEAM). In addition, the interventionist can consider administering an error analysis.

To customize the intervention, teachers should use the current and prior grade-level focus standards to determine the necessary components of the individualized plan. Teams will need to analyze all of the data available regarding a student (including the information from the formal diagnostic assessment and error analysis, if completed). Then a hypothesis must be developed about the underlying causes of the student's lack of progress so that a more individually customized intervention plan can be developed and implemented.

Step 6: Document Interventions

Different methods can be applied to keep these data (screening and progress monitoring) visible and usable. Charts are best for visual representations to help staff members interpret the progress monitoring data in relation to the student's goal. Assessment cards are an additional option for displaying both screening data and progress monitoring information to staff members. Whatever method of data display is used, it is important to ensure that data is maintained in a confidential manner but is readily available to staff members who work with the students.

Building Leadership Teams will also need to consider how individual student data will be shared with parents. Specific suggestions on how to share data with families can be accessed through the Kansas Parent Information Resource Center (KPIRC, www.ksdetasn.org/kpirc)

Interventions need to be logged once students are placed in appropriate groups. The student intervention log (sample here) and the progress monitoring graph need to be consistently updated so that an accurate record of the interventions and results can be maintained. It is critical that teachers document the instruction that they are providing, the intervention sessions that each student actually attends, and an accurate record of the progress monitoring results. This documentation is critical when analyzing student growth during consistent data review meetings during which instructional adjustments are made according to the team decision rules. This cycle of assessment, adjustment, and adding to the graph or log continues as long as a student requires intervention.

For students who continue to be non-responsive to interventions, it becomes critical to begin moving from a group problem-solving model to a more individualized format. The individual student problem-solving process is what schools have traditionally used for general education interventions, often conducted by student improvement teams (also known as SIT, SAT, TAT, and CARE teams, among other names). Within the Kansas MTSS and Alignment model, the collaborative teams conduct the work of the general education intervention team or student improvement team (SIT).

At any time, a leadership or collaborative team suspects a student may have an exceptionality, the team *must* refer the student for an initial evaluation. Any parent request for a special education evaluation must be reported to the building administrator or to the appropriate staff member designated by district special education procedures. Utilization of the Kansas MTSS framework should not delay a student from receiving a special education evaluation. A student does not have to move through all tiers before a referral for a special education evaluation is made. Conversely, having received all tiers of instruction or needing Tier 3 instruction does *not* solely indicate that a student should be referred for a special education evaluation.

When the Kansas MTSS framework is implemented, all parents must be informed of the nature of student performance data being collected, the general education services being provided, strategies for increasing a student's rate of learning, and parents' right to request an evaluation (K.A.R.91-40-10(f)(2)). Staff members and parents need to understand that a student may be referred for a special education initial evaluation when:

- The school has date-based documentation indicating general education interventions and strategies would be inadequate to address the areas of concern for the student or
- The school has data-based documentation that:
    - The student was provided appropriate instruction by qualified staff in regular education
    - The student was provided repeated assessment of academic achievement to demonstrate the student's progress during instruction.
    - The assessment results were shared with the parents.
    - The results indicated that an evaluation is appropriate (K.A.R.91-40-7(c)).

Math and Preschool MTSS

Long before young children enter school, they naturally and spontaneously explore and use mathematics. Starting in infancy, babies are curious about their world and begin to think about it in mathematical ways. At as early as 10 months of age, infants can distinguish a set of two items from a set of three. Unfortunately, mathematics has often taken a backseat to literacy, both in homes and in classrooms. When parents are asked which is more important, they more frequently say language and literacy over mathematics. They value early language and literacy skills because they are reflective of how children communicate and express themselves. Teachers of young children also tend to neglect teaching mathematics due to their own negative experiences with math (NRC 2009; Clements & Sarama, 2009; Copley 2010).

Nevertheless, learning mathematics is vital for young children's academic success. Not only does early mathematical competence influence children's future success in mathematics, it can also impact success in literacy, science, and technology. Early mathematic competence is actually one of the best predictors of school success across the curriculum (Duncan et al., 2007; NAEYC/NCTM, 2010).

Children's learning of mathematics is improved when children are provided research-based teaching and learning strategies, well-planned and sequenced curriculum, and integrated mathematical experiences (i.e., teaching mathematics through meaningful contexts). Young children need opportunities to practice and extend mathematical thinking through play, exploration, and creative thinking (NRC, 2009), which makes the application of the Kansas MTSS and Alignment for mathematics in preschool especially important.

The National Research Council (NRC) Committee on Early Childhood Mathematics along with the National Council of Teachers of Mathematics (NCTM) recommends that preschool mathematics instruction concentrate on three areas, also known as focal points: numbers and operations (i.e., numbers, comparison, counting, and cardinality), geometry (i.e., shapes and spatial relationships), and measurement (i.e., identifying measurable attributes and comparisons based on those attributes). NRC and NCTM also recommend prioritizing instructional time for each of these areas. The primary focus of mathematics instructional time in preschool should be on numbers and operations. The second priority should be geometry, but with less time devoted to it than numbers and operation. Thirdly, NRC and NCTM recommend thatonly a small amount of instructional time be set aside for measurement. In addition, they recommend that the work on patterning and data be woven into the main three focal areas and not taught using the same time investment as the prioritized topics. It should be noted that, when lessons combine more than one of the topics (e.g., including numbers in a geometry lesson), it facilitates learning and deepens the understanding of both topics (NRC, 2009; Fusion et al., 2010). The Kansas MTSS and Alignment was designed to utilize these key areas and early mathematical research to support all learners.

There is a need for more intentional and explicit mathematics instruction in preschool classrooms. Teaching all students to be mathematically competent requires a system for the early identification of students who are at risk as well as a system for providing those students with the interventions they need to become proficient in mathematics. Good classroom curriculum and instruction can meet the needs of most students; however, an efficient system for providing high quality interventions is required to ensure that the needs of all students are met.

At times, the application of the Kansas MTSS and Alignment in preschool will be slightly different from what might be put in place for school-aged children; however, the basic process and practices are similar. For appropriate application to occur, leadership teams must understand the similarities and differences between programming for very young children and those approaches used in more formal schooling. It is important that programs use evidence-based instructional practices that have been shown to be effective with young children, including developmentally appropriate teaching strategies.

Creating the Structure for a Preschool MTSS

The guidance for creating the necessary structure for a preschool MTSS and Alignment currently focuses on the following:

1. Implementation of an evidence-based core curriculum that supports the acquisition of early math skills and serves as the foundation for meeting the needs of *all* children.
2. Instructional strategies and interventions that support the acquisition of early math skills through differentiated instruction (e.g., small flexible groups, embedded learning opportunities).
3. Determination of preschool end-of-the-year learning targets based on information gathered from curriculum-based assessments, early math general screening tools, and/or other means (e.g., Kansas Early Learning Document: Early Learning Standards [KSELD]) as identified by your leadership team.
4. Universal screening and progress-monitoring activities that assess the areas of early math that are predictive for later math success, specifically those activities included in numbers and operations.
5. Identification of preschool children for whom the core curriculum and instruction does not appear to be sufficient and who might need more intensive instruction.
6. Provision of tiered support (Tier 2/3) through more targeted instruction on specific skills.

Tier 1/Curriculum and Instruction for Early Mathematics

From birth, young children develop knowledge and skills that lay a foundation for later mathematical ability. These skills do not develop in isolation; they are intertwined with other developmental domains (Copple & Bredekamp, 2009). As young children explore their world, specific interests spark in-depth investigations, and playtime provides meaningful opportunities to practice and become proficient. Preschool teachers must intentionally create environments and utilize instructional strategies to build children's math competence and conceptual knowledge, while also promoting the capacity for reflection, explanation, and justification of thinking.

How preschool educators teach math is as important as what they teach young children. Early mathematics experts advocate for a balanced approach to preschool instruction (NRC, 2009; Fuson et al., 2010). Traditionally, math in preschool has focused on narrow skill-based instruction embedded within calendar and/or center time, which often results in haphazard or random instruction. When teachers do give mathematics adequate curricular time, they often try to cover so many topics that the results can be superficial and uninteresting to children.

Strong early mathematical programs provide a combination of teacher-directed and child-initiated activities, differentiation, grouping strategies (large, small, and individual), and flexible schedules that allow for sustained and in-depth learning through play and responsive/nurturing teaching techniques (Clements & Sarama, 2009; NAEYC/NCTM, 2010). Teachers should teach specific math lessons in a logical sequence to expand children's learning to a deeper level of understanding and to ensure that no skills are left to chance. That same logical sequence must also be incorporated through the indirect teaching that happens across the daily schedule.

It is important to understand that mathematics and mathematical thinking are distinct in nature. Mathematics is a means of mathematical thinking and reasoning. Mathematical proficiency is one of the greatest predictors of the future academic success of students (Duncan et al., 2007).

The NRC and NCTM identified the following three areas as essential preschool math focal points:

- Numbers and Operations
- Geometry
- Measurement

To achieve depth of instruction, the majority of a preschool math curriculum's instructional time should be focused on teaching numbers and operations, followed by geometry, and finally measurement, rather than covering every topic or every skill with the same weight. While data analysis and algebra are also important, they should not be given the same amount of instructional time as the first three; instead, they should be woven into the first three (NRC, 2009; Fuson et al., 2010).

Numbers and Operations

Numbers and operations form the primary curricular focal point, and success in these areas should be the primary goal for instruction in preschool. These are the areas in which the most math instructional time should be spent. Quantity or number sense can be as important to math development as phonemic awareness is to emergent literacy. Just as children need to hear language, rhymes, and sounds for early literacy, they need experiences with numbers and operations to fully develop their number sense (NRC, 2009; Fuson et al., 2010; McCray, 2007).

Numbers and operations are more than simply recognizing numbers and counting; they also involve a conceptual understanding of numbers and what they represent. Numbers and operations are made up of three major components:

- Number Core
- Relations Core
- Operations Core

Number Core

This component is the largest element of numbers and operations and contains the most crucial skills for numerically proficient children. Those skills include cardinality, knowing the number word list, one-to-one correspondence, and written number symbols.

Cardinality is recognizing that the last number said is the number that represents the set of objects. Simply stated, it is knowing how many you counted. Often, children will count a set of items such as 6 buttons, then if a teacher picked up the last button she counted in the set, she might tell you that button is six. Children must learn to remove the labels of the number words from each individual item and gain an understanding that the last number they stated represents the entire collection, not just the last item. When first teaching cardinality, it is important to start with a small group of items arranged in a row, count from left to right, and make sure the items being counted are of similar size and shape.

Once children seem confident with cardinality, teachers can deepen their level of understanding by asking a child to start in the middle or at any point along the set of objects and count and tell you how many they have. This incorporates order irrelevance or an understanding that it doesn't matter which order in which you counted the set, there are still the same number of items. Finally, children need to learn object irrelevance or gain the understanding that items do not have to be the same size or shape to be part of a set that can be counted (Clements & Sarama, 2009).

The next skill included in the number core is **knowing the number word list**, often referred to as rote counting. Children need to learn that there is a stable order in which to say the number word list, and it is the same every time. By age 3, most children understand that numbers go in a certain order, but they will often skip numbers and don't have a good sense of the entire number word list. This happens because children learn the number word list as a string of words without meaning. Once they begin to attribute meaning to the words, you might notice that the children who once could count to 10 or 20 without skipping begin skipping numbers or getting them out of sequence. It can appear as if they have stepped backwards in their learning, but they haven't. They are making sense of the number words and how they fit together, and they will soon swing back to a stable count order and continue their growth (Brownell et al., 2014).

Once children can consistently count starting at zero or one, it is important to begin working with them to begin counting from any number and counting on. The ability to start at any given number and count onward is a precursor to addition and subtraction. This is a skill that cannot be taught or practiced too much. It is important to create fluidity and automaticity in counting so as to allow children's brains to focus on higher-level math skills. There are several steps involved in problem solving; if children have to think about the number word list, they are unable to perform more difficult counting tasks, including making comparisons between two groups and basic addition or subtraction. Multiple opportunities to practice can become more effective when paired with a movement for every word in the list, such as clapping, stomping, or jumping (Clements & Sarama, 2009). However, overemphasis on rote counting to high numbers before establishing counting principals such as cardinality and one-to-one correspondence with small numbers is counterproductive. The goal in preschool mathematics is to gain a deeper understanding of numbers and what they represent (Brownell et al., 2014).

One-to-one correspondence is another skill housed within numbers and operations. This is the understanding that each item you are counting gets one and only one count. This skill should be paired with cardinality to help children develop the understanding that the purpose of counting is to determine “how many.” Fluency in counting objects in combination with knowing the number word list is a crucial skill for children to move forward in higher-level mathematical concepts. The final piece of numbers and operations is the **written number symbol**. Children should be able to recognize the written numeral and understand that it is a symbol that represents a conceptual understanding of a set of four. The quantity of four is so much more than a “4” written on a page. As children develop meanings for the written numerals or number symbols, they should also compare these numerals with the quantities they represent. Using multiple representations for the quantity of each numeral helps to build conceptual understanding. This might look like matching the numeral to the time on the clock, to a pattern on a die, and to objects in a set (Fuson et al., 2010).

Relations Core

This component is about relationships, which can be determined by looking at attributes of a set. To support this skill in the classroom, it is important to provide not only visual comparisons but also multiple opportunities to build connections between those visual sets to number words and quantity terms (e.g., a group with five bears is more than a set with three). The greater the use of a variety of words to describe comparisons, the stronger the vocabulary children will acquire and be able to use in their own math talk. Preschool teachers tend to concentrate on the “bigger” attribute, such as which has more, which has the most, and which is longer. It is important to broaden children’s vocabulary and understanding by giving equal time to the concepts of less, smaller, fewer, and shorter (Clements & Sarama, 2004). To give children a real sense of number size and support their ability to make reasonable comparisons, include benchmark collections of sets of objects posted in your room, such as sets of 3, 10, 20, 50, and 100. When doing estimations, support students’ learning by teaching them to use those benchmark collections so they are not just guessing how much, but also to have an anchor to help them make educated predictions (Brownell, 2014).

Included within the relations core are the skills of **conservation of number and subitizing**. *Conservation of numbers* is another concept preschool-age children acquire. Children grow in their understanding that the number remains the same regardless of how items in sets are distributed or their likeness in size or shape. Prior to a child developing conservation of number, they tend to focus on their perceptions of things rather than the factual information. For example, when objects in two groups are a different size or shape, even after counting them, a child will perceive the group with the larger items to have more. The same is true if the items in one group are spread out, making a longer line. Once they gain conservation of number, children understand that there are the same number of items in a group, and they begin to consider the factual information of how many they counted over the perceptual information of how it looks. Providing multiple opportunities to count and recount the same number of items in different configurations and talking about their results help to solidify young children’s conservation of numbers (Fuson et al., 2007).

Subitizing involves seeing sets and knowing how many there are without physically counting every item. Subitizing introduces basic ideas of cardinality, or how many, as well as more and less, parts and whole, and quantity. It also has a direct link to addition and subtraction.

Subitizing is one of the main abilities young children should have opportunities to develop and can be an area that is lacking, especially among children from low-resource communities and those with special needs (Clements & Sarama, 2004). When helping children acquire the skill of subitizing, it is important to change the configurations of sets so children are not just memorizing a visual pattern, but truly gaining a sense of how many. Always start with linear patterns, followed by a domino or dice configuration, leaving random scatters for more advanced practice, as they are the most difficult for children to subitize. It is also important to use similar shapes on a contrasting background. Teachers should avoid using cute pictures and busy backgrounds that might prevent children from seeing the set clearly. Subitizing practice can be done with dots on a page or with simple manipulatives, such as lacer links or inch cubes on a table. Teachers should quickly show children different arrangements of items of one to five and ask them how many they saw. In addition to supporting an understanding of how many and quantity comparisons, subitizing also helps to develop an understanding that there are numbers within numbers. For instance, 5 can be made up of 3 and 2 or of 4 and 1. If we only present numbers through manipulatives in a straight line, children tend to live in "ones world" and do not see how numbers can be composed. As you begin to work with numbers larger than 4, it is important to emphasize the numbers within the numbers. If 7 is presented in a straight line of dots or manipulatives, the tendency is to count each dot or manipulative. However, if we present 7 as a 3 and a 4, we begin to encourage students to see numbers as compositions of other numbers. This is a critical element of quantity and a precursor to addition and subtraction (Clements & Sarama, 2009; Fuson et al., 2010).

Operations Core

The Operations Core deals with the addition and subtraction of objects. In preschool, when working on basic operations, students should always start with some type of concrete object. This also helps to put the problem in a story form and use physical objects related to the story. The movement involved when children use manipulatives or act out a story problem helps to anchor their learning and solidify their understanding of what it means to add and subtract. Fingers are a great manipulative as well. One can allow and even teach children to use their fingers when solving problems. There is no need to worry about fingers becoming a crutch for students. Once children gain a better understanding of mathematical processes later in life, this strategy is replaced by more efficient ones (Clements & Sarama, 2007). When children become proficient at using manipulatives through story problems, they can then move into drawings to solve their equations. Only at this point should they be introduced to written expressions and equations using appropriate terminology and symbols (+, –, and =). Having multiple words in their vocabulary for similar actions helps support their conceptual learning of the skill. When teaching addition, use a variety of terminology, such as *add, join, put together, plus, combine,* and *total*. Subtraction vocabulary includes *minus, take away, separate, difference*, and *compare.*

Geometry

Geometry is the second mathematic focal point determined by NRC and NCTM. While focused instructional time in geometry is needed at the preschool level, it is recommended that the time spent on these concepts be less than the time devoted to numbers and operations (Fuson et al., 2010). From the earliest years, children learn about shape and spatial relationships. At first, they are not able to distinguish circles, triangles, and squares from each other, but gradually they develop a richer sense of the parts and attributes of these shapes, along with the ability to orient them in a space (Fuson et al., 2010).

Young children should learn three components of geometry:

- Shape and structure
- Composition and decomposition of shapes
- Spatial reasoning

Shape and Structure

The shape and structure component focuses on teaching children to recognize two- and three-dimensional shapes. Children learn their understanding of shapes based on example; therefore, children will develop a more accurate sense of shape when they are exposed to a wide variety of shapes within each category. Children also need to be exposed to examples of shapes beyond circles, squares, rectangles, and triangles; otherwise, they will develop a limited understanding of shapes. For instance, children might not think of a trapezoid as a shape, because it’s not a shape that has a name they know. Forming accurate mental models of shapes is important, but not sufficient. When teaching children about shapes, it is also important to include vocabulary that helps them analyze shapes and understand that shapes are characterized and defined by certain parts or attributes. For example, a triangle has three sides and three corners or vertices, and a square has four sides of equal length. When teaching shapes and their attributes, make sure to include a wide variety of vocabulary that includes both common names and appropriate mathematical terms such as oval and ellipse, corners and vertices, and ball and sphere (Fuson et al., 2010; Brownell et al., 2014).

Composition and Decomposition of Shapes

Composition and decomposition of shapes helps children understand that shapes can be taken apart or put together to create other shapes. For instance, two triangles can be put together to create (compose) a square, or a square can be taken apart (decomposed) to create two triangles. Show children that shapes can also be transformed when they are used to fill other shapes or put together to create pictures, much like children do when using pattern blocks and picture cards. Symmetry is also an integral part of the composition and decomposition of shape. Symmetry includes flipping the same shape over, sliding it, or rotating it into new positions to duplicate designs and three-dimensional structures (Clements & Sarama, 2009).

Spatial Reasoning

Spatial reasoning includes two main abilities: spatial orientation (i.e., knowing where you are and how to get around) and spatial visualization (i.e., building and manipulating objects mentally). Children’s skills are initially based on their position within an environment, but they quickly expand to include external references. To teach spatial reasoning, it is important to include spatial vocabulary such as “on, in, under, and over” in your daily language with children and also to provide interesting environments for them to explore and navigate. When going to typical places (e.g., recess or the restroom) with young children, talk about the landmarks you see on the way (i.e., drinking fountains, pictures on the wall), or the routes you take to get to your destination. To deepen children’s understanding, encourage them to draw maps beginning with very familiar areas (e.g., their bedroom, their home, the playground, or the classroom) and then extend it to have them create maps of a city they built in the block area or a zoo they have created in dramatic play (Fuson et al., 2010).

Puzzles are another tool to teach spatial reasoning, especially spatial visualization. Children build these skills as they visualize the pieces and work to determine where a puzzle piece fits or how they might turn it to get it to fit. A great tool to increase the speed at which children acquire spatial reasoning and visualization skills are computer puzzles. This is one area in which computers can be more effective in preschool than hands-on manipulatives. A child might turn and manipulate a physical puzzle piece and not really pay attention to what they have done or how they moved it. On a computer, they have to be aware of the decisions they are making to put the piece in the appropriate spot. This process helps them articulate what they have done more effectively and solidifies their learning of spatial reasoning (Clements & Sarama, 2009).

Measurement

Children who are surrounded with interesting objects, such as blocks and sensory tables with a variety of containers, are naturally led to discover relationships among them and how they are the same or different. The more frequently children make comparisons, the more complex their comparisons become. Measurement develops from a need to compare two or more objects in a variety of ways. It has a way of bringing both geometry and numbers together as children explore and experiment with their comparison. When making comparisons with numbers, it is important to teach both sides of specific measurement terms (e.g., more and less, same and different, heavy and light, longer and shorter, full and empty) and to incorporate specific measurement terms into your daily vocabulary whenever possible.

While measurement is important, the NRC and NCTM recommend that only a small amount of instructional time be spent on this third focal area (Fuson et al., 2010). The focal area of measurement involves skills for identifying measurable attributes and making comparisons based on those attributes, including length, area, and volume.

As children begin to measure using non-standard units, teachers support their learning by helping them recognize the need to measure by using the same unit. Children often see no problem mixing non-standard units as they measure; for example, using both blocks and paperclips at the same time to cover what they are trying to measure. Children must learn that, when units are not equal, they are not units that can be used for measuring. Once children have measured something and are counting the units, they always need to clarify their units by labeling them and using the measurement terms, not just the number count. For example, teach them to say, “the flag is seven blocks long” instead of “the flag is seven.” Another common error children make is leaving gaps between the units instead on aligning them end to end. You can help children develop an understanding of the necessity to align the objects without gaps by comparing that child’s results with the results of another child who measured the same item with the same units without gaps. Initiate conversations about the varying results, helping them discover the reason for the different measurements (Brownell, 2014).

Once children become proficient at measurement using non-standard units, teachers can further their understanding by teaching them the relationship between the size and number of the nonstandard units used. This is done by comparing the results of measuring the same object multiple times using a variety of manipulatives. For example, when measuring the length of a table, have one child measure using inch cubes while another measures the same table using a block from the block area. Discuss the results and the children’s conclusions about why they came up with different number amounts in their measurement.

Finally, preschool is the perfect time to begin to support children in making indirect comparisons using a third object. For example, the children can dig two holes on the playground and use a stick to see which one is deeper (Clements & Sarama, 2004).

Algebra and Data Analysis

The foundations for data analysis in preschool lie in other areas such as counting and classifications. The calendar serves as a connection to the focal points of numbers and operations and geometry. As children learn to sort objects and quantify their groups, they also gather data to answer questions, classify those answers into categories, and quantify their responses (Clements & Sarama, 2009). Data analysis has deep connections to the other focal point areas; NCTM does not recommend giving data analysis its own block of instructional time, but rather recommends combining it with the other areas.

Algebra at the preschool level involves finding patterns in the world. However, the concept of pattern goes beyond the typical practice of making patterns in preschool classrooms, which involves creating simple sequential patterns such as red, blue, red, blue with manipulatives or on a paper chain. It includes seeing more complex patterns, such as the perceptual patterns found on dominoes, the patterns found in the number world, the repetition of numerals 0-9, and the pattern of one more when counting and adding (Clements & Sarama, 2009). It's important for teachers to understand patterning in all its forms, so they can take children beyond simple linear patterns to a way of thinking about patterns that support them in making mathematical connections. This is done by talking about patterns not only in terms of the ABAB pattern and its variations (e.g., AABAAAB and ABCABC), but also assigning numerical value to patterns, identifying when there are two of something and then one of something else, or noticing that every third one is yellow. These deeper conversations help children make mathematical generalizations (e.g., when adding a zero to a number, the sum is always that number, or when you add one more, the sum is always the next number in the number sequence). It is important to tie this type of algebraic thinking to both numbers and shapes.

Calendar

When thinking about preschool mathematics, many teachers immediately think about "calendar time" and believe it infuses math into the student's day. The calendar seemingly provides the time to teach patterning, numbers, and the number word sequence on a regular basis. However, scholars (Ethridge & King, 2005; Beneke, Ostrosky & Katz, 2008) have questioned the calendar as a context for preschool mathematics and the breadth/depth of the mathematical learning the children actually receive during a typical calendar routine. When reflecting on the value of incorporating calendar time into preschool, teachers must consider their instructional goals for this routine and whether the skills being taught and the strategies used to teach those skills are developmentally appropriate.

To participate meaningfully in calendar activities, young children must understand that time is sequential (i.e., yesterday, today, and tomorrow; morning, afternoon, and evening; Sunday, Monday, Tuesday, and so on). Children also must be able to conceptualize before and after and think about future and past events. While preschoolers can recall past events and talk about what will happen, most do not understand the concepts of yesterday, today, or tomorrow and are not able to talk about these events in terms of units of time, such as days or weeks. The ability to judge the relative time from a past event or until a future event typically is not in place until sometime between the ages of seven and ten (Beneke et al., 2008; Ethridge & King, 2005).

While children can learn to recite the days of the week and months of the year, they assign little meaning to them. Psychological time, which focuses on the important events in a child's life or the life of their family (i.e., birthdays, trips, and events), is much more relevant to young children than time on a clock or days on a calendar and does not correspond to regular units of minutes, hours, days, and months.

Early educators sometimes use the calendar to teach concepts other than time, such as sequencing, patterning, and number recognition. However, this too has its own set of complications. The seven-column, five-row grid of a calendar interferes with children's understanding of our base-ten system. The calendar grid gets filled in a different way each month, the day a month begins is different every month, and the number of days is variable; this prohibits children from seeing true patterns in the numbers. Seven is an unnatural break in our 0-9 sequence of digits and doesn't support children's use of their natural counting tools – their fingers (Brownell et al., 2014).

The 10-20 minutes spent on the calendar during a preschool day would be more productively spent on intentional math instruction, which gives preschool children opportunities to explore and experiment with math concepts, use concrete materials, and interact with a responsive adult to question and guide learning (Brownell, 2014; Beneke et al., 2008).

Standards and Curriculum

The Kansas Early Learning Standards (KELS) document provides a starting point for teachers and curriculum committees. The KELS document offers information and guidance to preschool providers on the developmental sequence of learning for children from birth through kindergarten. Aligned with the Kansas K-12 Standards, the KELS are structured around domains for learning that include a whole-child perspective.

The KELS were not designed to serve as an assessment or a curriculum. Rather, they were designed to guide educators in selecting curricula and assessments focused on the skills and the knowledge young children should have as a result of participating in high-quality preschool programs. An understanding of early math development, the curriculum focal points of early math, and evidence-based instructional strategies are fundamental considerations when selecting preschool mathematics curriculum materials.

The Kansas MTSS system of alignment advocates for the selection of a comprehensive, evidence-based preschool curriculum that addresses all domains of learning outlined in the Kansas Early Learning Standards. While your MTSS efforts are focused on academics and/or social behavior, when it comes to intervention, it is important that programs use curricula that address the needs of the whole child. Programs are encouraged to use resources such as the Head Start Preschool Consumer Reports and/or the What Works Clearinghouse to examine the evidence-base of different preschool curricula.

In addition, programs should examine their selected curriculum to determine whether the three focal points for mathematics instruction are adequately addressed. Some comprehensive curricula provide strong support for early mathematics, while others might not include all three essential areas with the appropriate weight. If this is the case, supplemental mathematic materials might be necessary to strengthen your overall program and ensure that students' outcomes are maximized.

Assessment

Comprehensive Assessment Plan and Data-Based Decision Making

Preschool programs already use a variety of assessment tools for a variety of purposes. Developmental screening tools (e.g., DIAL, ASQ) are used to determine which students might have developmental delays and need further assessment. Diagnostic assessments (e.g., Braken, Brigance, PLS, Peabody Motor Scales) often compare children to a standardized sample and are most generally used to determine whether a child might qualify for special education or other services. Curriculum-based assessments (e.g., AEPS, Carolina, Teaching Strategies Gold) are used multiple times per year to measure a child's progress over time and help teachers in planning core curriculum. Program assessments (e.g., ECO, Kindergarten Readiness Snapshot) are often measures required by funders and used to evaluate the overall effectiveness of programs. In the Kansas MTSS process, the first step in creating a comprehensive assessment plan is to consider the assessment tools you are already using, the purposes for which you are using those tools, and whether there are tools or practices that are duplicative in purpose or are no longer necessary. This information should be documented on your district's Comprehensive Assessment Plan along with other decisions your leadership team makes about the assessments that will be used in your program.

Universal Screening

The next step in the MTSS process is to determine what your program will use as a universal screening tool. Unlike developmental screening tools, a universal screening tool is used to compare students to a normative sample or standard for the purpose of identifying which students might be at risk for later learning difficulties based on indicators that are predictive of later achievement. A developmental screening tool identifies children who might have a developmental delay, while a universal screening tool identifies students who might be at risk and ranks them into levels/tiers based on that risk. This distinct difference makes the data from universal screening tools particularly helpful for examining the effectiveness of your curriculum and supports a process for tiered intervention.

Universal screening tools appropriate for early mathematics assess the skills related to number sense. Typically, these skills include counting, cardinality, number recognition, and quantity comparisons. They are valid and reliable for this purpose, can be used with confidence to make instructional decisions, and can be given at least three times per school year. Leadership teams must ensure that they have a tool that examines the predicative elements of early mathematics.

Creating a comprehensive assessment system is one of the major structuring tasks that must be completed by your leadership team. The Kansas MTSS and Alignment recommends screening preschool students at least three times per year using a universal screening tool. This information should be reviewed alongside the elementary universal screening data to support discussions related to the adequacy of your preschool curriculum, the match between your preschool and kindergarten scope and sequence, and the information necessary to meet the needs of individual students. When comparing preschool and elementary data, leadership teams should keep in mind the make-up of their preschool population and how it differs from the kindergarten population. In most school systems, not all kindergarten students attend a public preschool program. Additionally, the students who do attend preschool in a public school often had to qualify for that program because they met at-risk criteria or were receiving preschool special education services.

Your leadership team will use universal screening data to examine the adequacy of your curriculum and your system's need for professional development. Classroom staff members will use universal screening data to plan for differentiated instruction within the core curriculum to identify students in need of additional support for mathematics and to determine the focus of that intervention. Each universal screening tool sets the criteria for determining which students are at or above benchmark and which students need Tier 2/3 support. Programs should follow the decision rules for the tool they select when using this information to group students into levels of tiered support.

Progress Monitoring

Progress monitoring is conducted within the Kansas MTSS and Alignment to inform staff members of students' growth related to content knowledge and skills. Regular progress monitoring and use of the data when making instructional decisions results in students making more academic progress than when teachers do not use progress monitoring. Teachers' accuracy in judging student progress increases when progress monitoring strategies are used consistently (Stecker & Fuchs, 2000).

For preschool students in the core (Tier 1), progress monitoring is often conducted using curriculum-based assessments (e.g., AEPS, Teaching Strategies Gold), administered three to four times per year. These assessments are tied to content-area instruction and help teachers determine whether students have learned the concepts and skills taught so that the subsequent instruction can be adjusted to re-teach concepts or provide additional practice of skills not yet mastered.

For students receiving supplemental (Tier 2) and intensive (Tier 3) instruction, progress monitoring data is used to chart the growth of individual students regarding the skills being targeted in intervention. Progress monitoring for students receiving supplemental or intensive instruction addresses two questions:

1. Is the intervention working?
2. Does the effectiveness of the intervention warrant continued, increased, or decreased support?

Unlike the K-12 MTSS system, preschool universal screening tools generally cannot also be used as progress monitoring tools, because they cannot be given with enough frequency to monitor intervention effectiveness and make changes to the level of intervention a student receives. Instead, preschool programs are encouraged to use mastery monitoring strategies to assess and monitor the progress of students receiving tiered intervention. Mastery monitoring strategies are teacher designed and involve directly collecting data on a student's mastery of the specific skills being taught in intervention. Typically, changes to the level of tiered instruction a preschool student receives will only happen after each universal screening benchmark period; however, teachers can use the data they collect through mastery monitoring, and their knowledge of the child to make changes when the intervention efforts (including the intensity of embedded learning opportunities) does not seem to be effective or indicate whether a change is needed.

Collecting and graphing progress-monitoring data over a series of weeks will provide a visual pattern of skill acquisition for students receiving additional support. The Kansas MTSS and Alignment recommends that mastery monitoring data collection in preschool occur at least one time every two weeks for students receiving Tier 2 support and one time every week for students receiving Tier 3 support.

Diagnostic Assessments

It is not generally necessary for leadership teams to identify a formal diagnostic process to determine an instructional focus in preschool. The skills being assessed at the preschool level are often basic enough to not warrant deeper evaluation. In the K-12 MTSS and Alignment system, diagnostic assessments are used to help narrow down the focus for intervention. Preschool early math intervention will focus on the number core. Some published protocol interventions, if selected, do have informal assessments that can be used to place a student into the appropriate level of the program and could be used at the preschool level.

Tier 2/3

Grouping for Preschool Math Intervention

Preschool populations by their very nature include children with a wide variety of skill levels. Therefore, preschool daily schedules are designed to provide multiple opportunities for differentiated instruction along the developmental continuum. All children, including those needing support through Tiers 1, 2, and 3, should participate in the core mathematics curriculum with differentiation provided. Differentiation of core curriculum is considered an element of Tier 1 for all students.

When considering how to provide interventions for students needing Tier 2/3 support, the Kansas MTSS and Alignment recommends that preschool programs not follow the intervention models typically used in K-12 programs. Instead of grouping students across classrooms or bringing in someone the child does not know, Tier 2/3 intervention is ideally provided in a child's classroom by familiar adults. It is especially important for young children to develop positive and secure relationships with adults. Research suggests that preschool teacher-child relationships play a significant role in influencing young children's social and emotional development (Fox & Hemmeter, 2009). Therefore, children identified through universal screening as requiring more support should receive that support through additional small groups and/or embedded learning opportunities within their daily routine and play.

Using the decision rules determined by your universal screening tool, children needing additional instruction in key early math skills will be identified to participate in intervention focused on a comprehensive intervention that encompasses the multiple skills included in number sense.

Tier 2

The Kansas MTSS and Alignment recommends that classroom teams consider at least one of two approaches when designing early math intervention for individual students. Classrooms can use a combination of both approaches to meet the individual needs of their students.

The first option involves the design of an additional small group (e.g., three or four students, two to three times per week for 10 to 15 minutes). Students needing Tier 2 support would be assigned to an intervention group based on the universal screening tool. Small groups could be provided in a variety of ways in a preschool classroom. Interventionists might pull students for a short time during self-directed learning activities or during other flexible times of the day (arrival/opening activities, transitions, snack time, etc.). Times for intervention can also be built into the daily schedule. Adults can work with all of the students in small groups of varying sizes and purposes. Interventions should be selected from the district's Tier 2 Protocol.

For some students/classrooms, it might make more sense to use the evidence-based strategy of embedded learning opportunities to provide a student with distributed practice across the daily schedule on intervention targets. Therefore, another option is to design an intentional schedule that provides a student with frequent documented embedded learning opportunities on targeted skills. The key to this option is the documentation of who, what skills, how, and when the embedded learning opportunities will occur and a method to ensure that each student receives the specified opportunities to practice each day. For this approach, teams will narrow down the learning target to a small set of skills that can be embedded based on developmental progressions. The use of a matrix, with the daily schedule listed vertically and the activities listed horizontally, can allow teams to create a process for when, with whom, and how embedding will occur. The specific learning targets should be listed on each student's matrix, and a process to keep track of when the opportunities are provided should be documented.

To increase opportunities for practice, it is also recommended that, whichever approach is used, a learning center be intentionally designed based on early math targets. Classroom staff members should encourage students needing Tier 2 support to participate in this targeted center multiple times per week. These learning opportunities should be designed to complement and extend what was learned in intervention as well as other early math topics addressed in the core curriculum.

Tier 3

Students who are identified as needing Tier 3 early mathematics intervention require more intensive opportunities to learn early math skills. Recommendations for Tier 3 look similar to those in Tier 2, but the intensity of the intervention should be increased through more frequent and smaller groups.

One option for intervention at Tier 3 is small-group instruction. In contrast to Tier 2, the group size for students needing Tier 3 intervention should be decreased, and the frequency should be extended (e.g., one or two students, four to five times per week for 10 to 15 minutes) to provide students more intensive support. Students needing Tier 3 support are assigned to an intervention group based on the need identified by the universal screening tool, and interventions should be selected from the district's Tier 3 Protocol.

The use of embedded learning opportunities can be an especially useful strategy for some students needing Tier 3 early math intervention. Therefore, another option within Tier 3 is to design a schedule that provides a student with more frequent documented embedded learning opportunities with targeted skills. This option also requires documentation of how, with whom, and when the embedded learning opportunities will occur each day and a method to ensure that students receive the planned embedded learning opportunities each day.

To increase opportunities for practice, it is also recommended that instructors encourage students in Tier 3 support to participate in learning centers proactively designed based on early math targets multiple times per week.

Tier 2/3 Protocols

Leadership teams will develop a preschool integrated protocol that includes early numeracy. A protocol outlines a procedure or system of rules that govern the selection of intervention methods and materials based on the intervention area identified by the universal screening tool. Just as leadership teams determined the core curriculum, it is imperative that they consider what staff members will use to provide early math intervention. Protocols make it easier for staff members to implement interventions because they do not need to design individualized interventions for each student. It also helps leadership teams as they examine data. If teachers are selecting from the same few interventions and students are not making the expected progress, leadership teams have documentation that different intervention materials and approaches are needed.

Leadership teams should identify the current materials and critically evaluate them to ensure that the essential skills are represented and the materials support the targeted areas. Leadership teams must also consider the evidence base of different interventions and instructional approaches. Prior to selecting, purchasing, or using any instructional materials, it is critical to carefully review the research base and match it to the student population.

In the Kansas MTSS and Alignment, the intervention curriculum protocol incorporates a portion of the protocol methodology and the problem-solving model. This is referred to as a hybrid model. In a hybrid model, a set group of interventions is defined to be used throughout the system. The interventions are chosen from a list of evidence-based approaches designed for specific areas of concern. Collaborative teams determine which intervention is to be used first, based on their universal screening data. Once the intervention begins, progress-monitoring data is used to determine if an intervention needs to be adjusted, intensified, or customized, based on pre-established decision rules (McCook, 2006). Once the curriculum protocols are developed, leadership teams should determine a management system for organizing and using the materials selected to ensure that all staff members providing supplemental and intensive intervention know where materials are located and how they are organized, thereby allowing for efficient planning for instruction.

The goal of interventions should always be to accelerate learning. If student performance indicates that this is not happening, the intervention needs to be adjusted. “If instructional groups are too large, instruction is not properly paced or focused, or too many intervention sessions are cancelled, then impacts on student performance will be reduced” (Torgesen, 2006, p. 4). According to Torgesen (2006, p.4), one of the biggest risks of intervention groups is that we begin to expect a lower standard of performance for students who require them. For intervention groups to work properly, intervention systems require school-level monitoring and regular adjustments. This is accomplished in the Kansas MTSS and Alignment through collaborative teams who meet on a regular basis to analyze students’ progress, adjust instruction, and use the self-correcting feedback loop for communication. At least eight key aspects are involved in developing and maintaining an effective intervention system (Torgesen, 2006):

1. Strong motivation on the part of teachers and school leaders to be relentless in their efforts to leave no child behind.
2. A psychometrically reliable system for identifying students who need interventions in order to make normal progress in learning math.
3. A reliable system for monitoring the effectiveness of interventions.
4. Regular team meetings and leadership to enforce and enable the use of data to adjust interventions as needed.
5. Regular adjustments to interventions based on student progress. The most frequent adjustments should involve group size and time (intensity).

6. Enough personnel to provide the interventions with sufficient intensity.
7. Programs and materials to guide the interventions that are consistent with evidence-based research.
8. Training, support, and monitoring to ensure that intervention programs are implemented with high fidelity and quality.

Conclusion

Structuring and implementing the components of the Kansas MTSS and Alignment framework within a district is a complex and long-term process. While many details have been discussed throughout this guide, educators can also visit the Kansas MTSS and Alignment Mathematics Repository and the Kansas MTSS & Alignment Preschool Repository for a wealth of additional resources and guidance. Contact information for all regional Kansas MTSS Math and Pre-K Trainers are listed on each repository.

References

Agarwal, P. K., McDaniel, M. A., McDermott, K. B., & Roediger, H. L. (2020). *How to Use Retrieval Practice to Improve Learning.* Retrieved from: http://pdf.retrievalpractice.org/Retrieval PracticeGuide.pdf

Archer, A., & Hughes, C. (2011). *Explicit Instruction: Effective and Efficient Teaching*. NY: Guilford Publications.

Ashlock, R. (2006). *Error Patterns in Computation: Using error patterns to improve instruction* (9th ed.). Upper Saddle River, NJ: Pearson Education Inc.

Ashman, G. (2023). *Cognitive load theory*. Corwin.

Ashman, G. (2021). *The power of explicit teaching and direct instruction*. Corwin.

Beneke, S., Ostrosky, M., & Katz, L. (2008) Calendar time for young children, good intentions gone awry. Young Children, May 2008.

Brownell, J. O., Chen, J., Ginet, L., Hynes-Berry, M., Itzkowich, R., Johnson, D. M. S., & McCray, J. (2014) Big ideas of early mathematics: What teachers of young children need to know. Boston, MA: Pearson and The Early Math Collaborative: Erickson Institute.

Bryk, A. S. et al. (2010) Organizing schools for improvement. *Phi Delta Kappan* (91), 7. Retrieved from: https://nieer.org/wp-content/uploads/2015/07/Organizing-schools-for-improvemnt-Bryk.pdf

Buongiovanni, L. (Host). (2021). *The passion and science of math with Amanda VanDerHeyden.* [Audio podcast].Vurbl. Retrieved from: https://vurbl.com/listen/3M2qBO55MTm/

Carta, J. J., & Young, R. M. (2019). Multi-tiered systems of support for young children: Driving change in early education. Paul H. Brookes Publishing Co.

Carpenter, S. K., & Agarwal, P. K. (2020). *How to use spaced retrieval practice to boost learning*. Retrieved from: http://www.retrievalpractice.org

Clements, D. H., & Sarama, J. (Eds.) (2004). Engaging young children in mathematics; Standards for early childhood mathematics education. Mahwah, N.J.: Lawrence Erlbaum Associates.

Clements, D. H., & Sarama, J. (2009). Learning and teaching early math: The learning trajectories approach. New York, New York: Routledge.

Codding, R. S., Volpe, R. J., & Poncy, B. C. (2017). *Effective math interventions: A guide to improving whole-number knowledge.* New York, NY: Guilford Press.

Common Core State Standards Math (n.d.). National Governors Association Center for Best Practices and Council of Chief State School Officers. Retrieved from: http://www.corestandards.org/Math/Practice/

Copley, J. (2010). The young child and mathematics 2d ed. Washington, DC: National Association for the Education of Young Children

Copple, C., & Bredekamp, S. (Eds.) (2009). Developmentally appropriate practice in early childhood programs serving children from birth through age eight. Washington D.C: National Association for the Education of Young Children.

Datchuk, S. M. & Hier, B. O. (2019). Fluency Practice: Techniques for Building Automaticity in

Foundational Knowledge and Skills, *Teaching Exceptional Children, Vol. 51* (6), pp. 424-435. DOI: 10.1177/0040059919847213

Dickson, S. V., Chard, D. J., & Simmons, D. C. (1993). An integrated reading/writing curriculum: A focus on scaffolding. *LD Forum, 18*(4), 12-16.

Donohoo, J. (2017). *Collective Efficacy: How Educators' Beliefs Impact Student Learning*. Thousand Oaks, CA, Corwin Publisher.

Duncan, G.J., Dowsett, C.J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., Japel, C. (2007). School readiness and later achievement. Developmental Psychology, 43, 1428-1446.

Ethridge, E. A., & King, J. R. (2005). Calendar math in preschool and primary classrooms: Questioning the curriculum. Early Childhood Education Journal, 32(5).

Fox, L., & Hemmeter, M. L. (2009). A program-wide model for supporting social emotional development and addressing challenging behavior in early childhood settings. In W. Sailor, G., Dunlap, G. Sugai, G., & Horner, R. (Eds.). Handbook of Positive Behavior Support. New York: Springer. 177-202.

Fuchs, L. S., Newman-Gonchar, R., Schumacher, R., Dougherty, B., Bucka, N., Karp, K. S., Woodward, J., Clarke, B., Jordan, N. C., Gersten, R., Jayanthi, M., Keating, B., and Morgan, S. (2021). *Assisting Students Struggling with Mathematics: Intervention in the Elementary Grades* (WWC 2021006). Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education. Retrieved from http://whatworks.ed.gov/

Fuson, K. C., Clements, D. H., & Beckman, S. (2010). Focus in prekindergarten: Teaching with curriculum focal points. Reston, VA: The National Council of Teachers of Mathematics, Inc.

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009a). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools* (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies. ed.gov/ncee/wwc/publications/practiceguides/

Gersten, R., Chard, D., Jayanthi, M., Baker, S., Murphy, P., & Flojo, J. (2009b). Mathematics instruction for students with learning disabilities: A meta-analysis of instructional components. *Review of Educational Research, 79*(3), 1202-1242.

Gersten, R., Clarke, B. S., Jordan, N. C. (2007). *Screening for mathematics difficulties in K-3 students.* Portsmouth, NH: RMC Research Corporation, Center on Instruction.

Gersten, R., Compton, D., Connor, C. M., Dimino, J., Santoro, L., Linan-Thompson, S., and Tilly, W. D. (2008). *Assisting students struggling with reading: Response to Intervention and multi-tier intervention for reading in the primary grades. A practice guide*. (NCEE 2009-4045). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies.ed.gov/ncee/wwc/ publications/practiceguides/

Hamilton, C., & Amador, O. (2019). *Scaffolding in a VISIBLE LEARNING Classroom.* Retrieved from https://corwin- connect.com/2019/02/scaffolding-in-a-visible-learning-classroom/

Haring, N. G., Lovitt, T. C., Eaton, M. D., & Hansen, C. L. (1978). *The fourth R: Research in the* *classroom*. Columbus, OH: Charles E. Merrill Publishing Co.

Hartman, J. R., Hart, S., Nelson, E. A., & Kirschner, P. A. (2023). Designing mathematics standards in agreement with science. *International Electronic Journal of Mathematics Education, 18*(3), em0739.https://doi.org/10.29333/iejme/13179

Hattie, J. A. & Donoghue, G. M. (2016) Learning Strategies: A synthesis and conceptual model. *Nature/NPJ: Science of Learning*. doi: 10:1038/npjscilearn.2016.13

Hosp, M., Hosp J., & Howell, K. (2007). *The ABCs of CBM: A practical guide to curriculum-based measurement.* New York, NY: Guilford Publications.

Kansas State Department of Education (2019). *Kansas Special Education Process Handbook*. Kansas Special Education Process Handbook (ksde.org).

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). Washington, DC: National Academy Press. National Research Council (2009). *Mathematics learning in early childhood: Paths toward excellence and equity*. Washington, DC: The National Academies Press.

Kirschner, P. A. (2020). *How learning happens: Seminal works in educational psychology and what they mean in practice*. Taylor & Francis Group.

Kirschner, P. A., et al. (2022). *How Teaching Happens: Seminal Works in Teaching and Teacher Effectiveness and What They Mean in Practice*. Routledge.

McCook, J. E. (2006). *The RTI guide: Developing and implementing a model in your schools.*

McCray, J. (2007). PCK for preschool mathematics: Teacher knowledge and math- related language contribute to children's learning. Chicago, Illinois.

National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM). (2010). Promoting good beginnings. A joint position of NAEYC and NCTM.

National Assessment of Education Progress (2022). *National Report Card: Mathematics.* Retrieved from https://www.nationsreportcard.gov/mathematics/nation/achievement/?grade=4

National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM) (2010). *Promoting good beginnings. A joint position of NAEYC and NCTM.*

National Association of School Psychologists (2020). *Class-wide Math Intervention Protocol*. Retrieved from: https://www.nasponline.org/resources-and-publications/resources-and-podcasts/covid-19-resource-center/return-to-school/class-wide-math-intervention-protocol

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for mathematics in prekindergarten through grade 8 mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2007). *Benefits of Formative Assessment.* Retrieved from: https://www.nctm.org/Research-and-Advocacy/Research-Brief-and-Clips/Benefits-of-Formative-Assessment/

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: National Council of Teachers of Mathematics.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education.

National Research Council (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, DC: The National Academies Press. https://doi.org/10.17226/9822

National Research Council. (2009). Mathematics learning in early childhood: Paths toward excellence and equity. Washington, DC: The National Academies Press.

National Academies Press. Retrieved from: https://doi.org/10.17226/9822

National Research Council (2009). *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*. Washington, DC: The National Academies Press. Retrieved from: https://doi.org/10.17226 /12519

Nelson, G., Pfannenstiel, K. H., Edmonds, R. Z. (2019). Examining the Alignment of Mathematics Instructional Practices and Mathematics Vocabulary between Core and Intervention Materials*. Learning Disabilities Research & Practice*, *35*(1), 14-24.

Nelson, P.M., Van Norman, E. R., Parker, D.C., Cormier, C.C., (2019). An Examination of Interventionist Implementation Fidelity and Content Knowledge as Predictors of Math Intervention Effectiveness. *Journal of Applied School Psychology, 35*(3), 234-256, doi: 10.1080/15377903.2019.1568334

Newman-Gonchar, R., Clarke, B., & Gersten, R. (2009). *A summary of nine key studies: Multi-tier intervention and response to interventions for students struggling in mathematics.* Retrieved May 25, 2010, from Center on Instruction: http://www.centeroninstruction.org/files/Summary%20of%209%20studies%20on%20 RTI%20math%20and%20 struggling%20 math%20students.pdf

Powell, S. R. & Nelson, G. (2016). *Trajectories of Mathematics Performance: From Preschool to Postsecondary.*

Powell, S. R., & Stecker, P. M. (2014). Using Data-Based Individualization to Intensify Mathematics Intervention for Students with Disabilities. *Teaching Exceptional Children, 46,* 31-37, DOI: 10:1177/0040059914523735

Programme for International Student Assessment. (2015). *Results in Focus.* Retrieved from: https://www.oecd.org/pisa/pisa-2015-results-in-focus.pdf

Ragsdale, J. (2020). *Maintenance Practice in the Mathematics Classroom* -Season 2 Episode 35. Retrieved from: https://www.youtube.com/watch?v=1-Tc3xP1i6Q.

Riccomini, P. (2005). Identification and remediation of systematic error patterns in subtraction. *Learning Disability Quarterly, 28*(3), 1-10.

Riccomini, P., & Witzel, B. (2010). *Response to intervention in math*. Thousand Oaks, CA: Corwin Press.

Rich, S. E. H., Duhon, G. J. & Reynolds, J. (2017). Improving the Generalization of Computer-Based Math Fluency Building Through the Use of Sufficient Stimulus Exemplars. *Journal of Behavioral Education, 26***,** 123–136. Retrieved from: https://doi.org/10.1007/s10864-016-9262-3

Rohrer, D., Dedrick, R. F., Agarwal, P. K. (2017). *Interleaved Mathematics Practice: Giving Students a Chance to Learn What They Need to Know*. Retrieved from: http://uweb.cas.usf.edu/~drohrer/pdfs/Interleaved_Mathematics_Practice_Guide.pdf

Sides, J. D. & Cuevas, J. (2020). Effect of Goal Setting for Motivation, Self-Efficacy, and Performance in Elementary Mathematics. *International Journal of Instruction. 13*(4): 1-16. DOI: 10.29333/iji.2020. 1341a.

Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Thompson, L., & Wray, J. (2010). Developing effective fractions instruction for kindergarten through 8th grade: A practice guide (NCEE #2010-4039). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://hatworks.ed.gov

Son, L. K., Brittingham-Furlonge, N., & Agarwal, P. K., (2020). *Metacognition: How to Improve Students' Reflections on Learning*. Retrieved from: http://pdf.retrievalpractice.org/MetacognitionGuide.pdf

Stecker, P., & Fuchs, L. (2000). Effecting superior achievement using curriculum-based measurement: The importance of individual progress monitoring. *Learning Disabilities Research and Practice*, 128-134.

The Science of Math. (n.d.). https://www.thescienceofmath.com/

The U.S. News/Raytheon STEM Index Shows America Will Have to Depend on Foreign Workers to Fill STEM Jobs. (2016, May 17). [web log]. Retrieved from: https://www.usnews.com/info/blogs/ press-room/articles/2016-05-17/the-us-news-raytheon-stem-index-shows-america-will-have-to-depend-on-foreign-workers-to-fill-stem-jobs.

Tomlinson, C. (1999). *The differentiated classroom: Responding to the needs of all learners*. Alexandria, VA: Association for Supervision and Curriculum Development.

Tomlinson, C. A. (2014) The differentiated classroom: Responding to the needs of all learners. 2nd Edition, ASCD, Alexandria.

Torgesen, J. K. (2006). Intensive reading interventions for struggling readers in early elementary school: A principal's guide. Portsmouth, NH. RMC Research Corporation, Center on Instruction.

Trends in International Mathematics and Science Studies (2019). *U.S. Highlights Web Report (NCES 2021-021).* U.S. Department of Education. Institute of Education Sciences. National Center for Education Statistics. Retrieved from: https://nces.ed.gov/timss/results19/index.asp

U.S. Department of Education (2021). Institute of Education Sciences. *Assisting Students Struggling with Mathematics Intervention in the Elementary Grades.* https://ies.ed.gov/ncee/wwc/PracticeGuide/26

VanDerHeyden, A., & Codding, R. (2020). Belief-Based Versus Evidence-Based Math Assessment and Instruction: What School Psychologists Need to Know to Improve Student Outcomes. Research-Based Practice; *Communique, 48*(5), 20-25.

Wilkerson, T. (2020). *Believing Our Students Can Do Mathematics: Identity, Agency, and Authority.* National Council of Teachers of Mathematics. President's Message. Retrieved July 9, 2021 from: https://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/ Trena-Wilkerson/Believing-Our-Students-Can-Do-Mathematics_-Identity,-Agency,-and-Authority/

Zakaria, E., Solfitri, T., Daud, Y., Zainal Abidin, A. (2013). Effect of Cooperative Learning on Secondary School Students Mathematics Achievement. *Creative Education*, *4*(2), 98-100.